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# Short Equational Bases for Ortholattices: Proofs and Countermodels

by

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# Short Equational Bases for Ortholattices: Proofs and Countermodels

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**Abstract**

This document contains proofs and countermodels in support of the paper "Short Equational Bases for Ortholattices", by the same set of authors. In that paper, short single axioms for ortholattices, orthomodular lattices, and modular ortholattices are presented, all in terms of the Sheffer stroke. The ortholattice axiom is the shortest possible. Other equational bases in terms of the Sheffer stroke and in terms of join, meet, and complement are presented. Computers were used extensively to find candidates, reject candidates, and search for proofs that candidates are single axioms.

## 1 Introduction

This report supports the (primary) paper "Short Equational Bases for Ortholattices" and the Web page *Short Equational Bases for Ortholattices: Web Support* [9]. The primary paper is essentially an extended abstract intended for a wide audience, the Web page is intended for immediate support, and this report is intended for archival support.

The overall structure of the three documents is similar, with the same section titles (even when the sections are empty). The primary paper contains no proofs, countermodels, or filters. The Web page contains proofs, countermodels, filters, and information on the software and methods used. This report contain the proofs, countermodels, and filters.

## 2 Equational Bases

First we define a chain of varieties from lattices to Boolean algebra in terms of join, meet, and complement. Then we go from ortholattices to Boolean algebra in terms of the Sheffer stroke.

### 2.1 In Terms of Join/Meet/Complement

This section contains multiequation bases, in terms of join, meet, and complement, for lattices ($\mathcal{L}$), ortholattices ($\mathcal{OL}$), orthomodular lattices ($\mathcal{OML}$), modular ortholattices ($\mathcal{MOL}$), and Boolean algebra ($\mathcal{BA}$). This development is not far from a standard development; and in the cases where it does depart, we prove the corresponding theorems and other theorems that may be of interest. For example, our basis for lattice theory does not contain commutativity, so we prove it; and we use a nonstandard modularity axiom, so we prove it equivalent (given

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lattice theory) to the standard axiom. Note that each of the bases for $\mathcal{OL}$, $\mathcal{OML}$, $\mathcal{MOL}$, and $\mathcal{BA}$ contains the equation $x \wedge y = c(c(x) \vee c(y))$, which allows us, if we wish, to rewrite the bases to be in terms of join and complement only.

Also, with the exception of $\mathcal{BA}$, we prove independence of the bases by giving countermodels found by Mace2. Many of the independence proofs are trivial, and we could give arguments without the help of Mace2, but we have included the Mace2 countermodels to emphasize the "automatic" nature of the work.

### 2.1.1 Lattice Theory

The following is a 4-basis for lattice theory ($\mathcal{L}$) in terms of join ($\vee$) and meet ($\wedge$).

| | |
|---|---|
| $x \vee (y \vee z) = y \vee (x \vee z)$ | % AJ |
| $x \wedge (y \wedge z) = y \wedge (x \wedge z)$ | % AM |
| $x \vee (x \wedge y) = x$ | % B1 |
| $x \wedge (x \vee y) = x$ | % B2 |

Note that this basis is self-dual; that is, the dual of each member is also a member.

A more standard basis for lattice theory is the 6-basis (also self-dual) consisting of B1, B2, and the commutativity and associativity of meet and join. Deriving commutativity of join (or of meet) from the 4-basis above shows that it is equivalent to the 6-basis. The following is an Otter proof of commutativity (idempotence is also proved).

**Proof LT1**

| | | |
|---|---|---|
| 1 | $A \vee B \neq B \vee A \mid \$Ans(CJ)$ | [] |
| 2 | $A \vee A \neq A \mid \$Ans(IJ)$ | [] |
| 4 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [] |
| 5 | $x \wedge (y \wedge z) = y \wedge (x \wedge z)$ | [] |
| 6 | $x \vee (x \wedge y) = x$ | [] |
| 8 | $x \wedge (x \vee y) = x$ | [] |
| 12 | $x \vee (y \vee (x \wedge z)) = y \vee x$ | [6 → 4] |
| 14 | $x \wedge x = x$ | [6 → 8] |
| 18 | $x \vee x = x$ | [8 → 6] |
| 20 | $\$Ans(IJ)$ | [18,2] |
| 21 | $x \wedge (y \wedge x) = y \wedge x$ | [14 → 5] |
| 74 | $x \vee (y \wedge x) = x$ | [21 → 6] |
| 169 | $x \vee y = y \vee x$ | [74 → 12] |
| 170 | $\$Ans(CJ)$ | [169,1] |

The following two Mace2 jobs show independence of the $\mathcal{L}$ 4-basis {AJ, AM, B1, B2}.

**Countermodel LTa**

```
x ^ (y ^ z) = y ^ (x ^ z).     % AM
x v (x ^ y) = x.               % B1
x ^ (x v y) = x.               % B2
A v (B v C) != B v (A v C).    % denial of AJ
```

C: 0 B: 0 A: 1

| $\wedge$: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

| $\vee$: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |

Table 1: LTa.out

**Countermodel LTb**

```
x v (y v z) = y v (x v z).    % AJ
x ^ (y ^ z) = y ^ (x ^ z).    % AM
x ^ (x v y) = x.              % B2
A v (A ^ B) != A.             % denial of B1
```

B: 0 A: 1

| $\vee$: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |

| $\wedge$: | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

Table 2: LTb.out

### 2.1.2 Ortholattices

The following is a 5-basis for ortholattices ($\mathcal{OL}$) in terms of join ($\vee$), meet ($\wedge$), and complement ($c$).

| | |
|---|---|
| $x \vee (y \vee z) = y \vee (x \vee z)$ | % AJ |
| $x \vee (x \wedge y) = x$ | % B1 |
| $x \wedge y = c(c(x) \vee c(y))$ | % DM |
| $c(c(x)) = x$ | % CC |
| $x \vee c(x) = y \vee c(y)$ | % ONE |

If we start with the lattice theory 4-basis {AJ, AM, B1, B2} and restrict that to the ortholattices by adding {DM, CC, ONE}, then AM and B2 become dependent. The following are Otter proofs.

**Proof OL1**

| | | |
|---|---|---|
| 1 | $x = x$ | [] |
| 2 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [] |
| 6,5 | $x \wedge y = c(c(x) \vee c(y))$ | [] |
| 8,7 | $c(c(x)) = x$ | [] |
| 10 | $A \wedge (B \wedge C) \neq B \wedge (A \wedge C) \mid \$Ans(AM)$ | [] |
| 11 | $c(c(B) \vee (c(A) \vee c(C))) \neq c(c(A) \vee (c(B) \vee c(C))) \mid \$Ans(AM)$ | [10,6,6,8,6,6,8] |
| 45 | $c(c(A) \vee (c(B) \vee c(C))) \neq c(c(A) \vee (c(B) \vee c(C))) \mid \$Ans(AM)$ | [2 $\rightarrow$ 11] |
| 46 | $\$Ans(AM)$ | [45,1] |

**Proof OL2**

| | | |
|---|---|---|
| 1 | $x = x$ | [] |
| 3 | $x \vee (x \wedge y) = x$ | [] |
| 6,5 | $x \wedge y = c(c(x) \vee c(y))$ | [] |
| 8,7 | $c(c(x)) = x$ | [] |

| | | |
|---|---|---|
| 10 | $A \wedge (A \vee B) \neq A \mid \$Ans(B2)$ | [] |
| 11 | $c(c(A) \vee c(A \vee B)) \neq A \mid \$Ans(B2)$ | [10,6] |
| 12 | $x \vee c(c(x) \vee c(y)) = x$ | [3:6] |
| 33 | $x \vee c(c(x) \vee y) = x$ | [7 → 12] |
| 50,49 | $c(x) \vee c(x \vee y) = c(x)$ | [7 → 33] |
| 53 | $A \neq A \mid \$Ans(B2)$ | [11:50,8] |
| 54 | $\$Ans(B2)$ | [53,1] |

The following Mace2 jobs show that the $\mathcal{OL}$ 5-basis {AJ, B1, DM, CC, ONE} is independent.

**Countermodel OLa**

```
x v (x ^ y) = x.              % B1
x ^ y = c(c(x) v c(y)).       % DM
c(c(x)) = x.                  % CC
x v c(x) = y v c(y).          % ONE
A v (B v C) != B v (A v C).   % denial of AJ
```

C: 0 B: 0 A: 1

| c: | 0 | 1 | 2 |
|---|---|---|---|
| | 0 | 2 | 1 |

| ∧: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 |
| 2 | 2 | 0 | 2 |

| ∨: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 |
| 2 | 2 | 0 | 2 |

Table 3: OLa.out

**Countermodel OLb**

```
x v (y v z) = y v (x v z).    % AJ
x ^ y = c(c(x) v c(y)).       % DM
c(c(x)) = x.                  % CC
x v c(x) = y v c(y).          % ONE
A v (A ^ B) != A.             % denial of B1
```

B: 0 A: 1

| c: | 0 | 1 |
|---|---|---|
| | 0 | 1 |

| ∨: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |

| ∧: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |

Table 4: OLb.out

**Countermodel OLc**

```
x v (y v z) = y v (x v z).    % AJ
x v (x ^ y) = x.              % B1
c(c(x)) = x.                  % CC
x v c(x) = y v c(y).          % ONE
A ^ B != c(c(A) v c(B)).      % denial of DM
```

B: 0 A: 0

| c: | 0 | 1 |
|---|---|---|
| | 1 | 0 |

| $\vee$: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

| $\wedge$: | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |

Table 5: OLc.out

**Countermodel OLd**

```
x v (y v z) = y v (x v z).    % AJ
x v (x ^ y) = x.              % B1
x ^ y = c(c(x) v c(y)).       % DM
x v c(x) = y v c(y).          % ONE
c(c(A)) != A.                 % denial of CC
```

A: 0

| c: | 0 | 1 | 2 |
|---|---|---|---|
| | 1 | 2 | 1 |

| $\vee$: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 |
| 2 | 0 | 1 | 2 |

| $\wedge$: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 2 | 2 | 2 |
| 1 | 2 | 1 | 2 |
| 2 | 2 | 2 | 2 |

Table 6: OLd.out

**Countermodel OLe**

```
x v (y v z) = y v (x v z).    % AJ
x v (x ^ y) = x.              % B1
x ^ y = c(c(x) v c(y)).       % DM
c(c(x)) = x.                  % CC
A v c(A) != B v c(B).         % denial of ONE
```

B: 0 A: 1

| c: | 0 | 1 | 2 |
|---|---|---|---|
| | 2 | 1 | 0 |

| $\vee$: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 2 | 0 | 1 | 2 |

| $\wedge$: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 1 | 2 |
| 1 | 1 | 1 | 2 |
| 2 | 2 | 2 | 2 |

Table 7: OLe.out

### 2.1.3 Orthomodular Lattices

The following is a 4-basis for orthomodular lattices ($\mathcal{OML}$) in terms of join ($\vee$), meet ($\wedge$), and complement ($c$).

$x \vee (y \vee z) = y \vee (x \vee z)$ % AJ
$x \vee (x \wedge y) = x$ % B1
$x \wedge y = c(c(x) \vee c(y))$ % DM
$x \vee (c(x) \wedge (x \vee y)) = x \vee y$ % OM

If we add the orthomodular law OM to the ortholattice 5-basis {AJ, B1, DM, CC, ONE}, then CC and ONE become dependent. The following is an Otter proof.

**Proof OML1**

| | | |
|---|---|---|
| 1 | $c(c(A)) \neq A \mid \$Ans(CC)$ | [] |
| 2 | $A \vee c(A) \neq B \vee c(B) \mid \$Ans(ONE)$ | [] |
| 4 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [] |
| 5 | $x \vee (x \wedge y) = x$ | [] |
| 8,7 | $x \wedge y = c(c(x) \vee c(y))$ | [] |
| 9 | $x \vee (c(x) \wedge (x \vee y)) = x \vee y$ | [] |
| 10 | $x \vee c(c(c(x)) \vee c(x \vee y)) = x \vee y$ | [9,8] |
| 12 | $x \vee c(c(x) \vee c(y)) = x$ | [5:8] |
| 17,16 | $x \vee c(c(x)) = x$ | [12 → 12] |
| 21,20 | $x \vee (y \vee c(c(x))) = y \vee x$ | [16 → 4] |
| 27,26 | $x \vee c(c(x) \vee y) = x$ | [10 → 12] |
| 32 | $x \vee (y \vee c(c(x) \vee z)) = y \vee x$ | [26 → 4] |
| 34 | $x \vee c(y \vee c(x)) = x$ | [20 → 26] |
| 36 | $x \vee c(c(c(x)) \vee c(y \vee x)) = y \vee x$ | [20 → 10,21] |
| 44 | $c(x) \vee c(x) = c(x)$ | [16 → 34] |
| 67,66 | $c(c(x)) \vee x = x$ | [44 → 20,17] |
| 76 | $c(c(c(c(x)))) \vee x = x$ | [66 → 20,17] |
| 84 | $c(c(x)) \vee (y \vee x) = y \vee x$ | [66 → 4] |
| 91,90 | $c(c(c(x))) = c(x)$ | [76 → 26,67] |
| 92 | $c(c(x)) \vee c(c(c(x)) \vee c(x)) = x$ | [76 → 10,91,91,91,91,67] |
| 115,114 | $c(c(x)) \vee c(y \vee c(x)) = c(c(x))$ | [90 → 34] |
| 119,118 | $c(c(x)) = x$ | [92:115] |
| 120 | $\$Ans(CC)$ | [118,1] |
| 127 | $x \vee (y \vee x) = y \vee x$ | [84:119] |
| 143 | $x \vee c(x \vee c(y \vee x)) = y \vee x$ | [36:119] |
| 153 | $c(x) \vee c(x \vee y) = c(x)$ | [118 → 26] |
| 170 | $c(c(x) \vee y) \vee x = x$ | [26 → 127,27] |
| 203 | $c(x \vee y) \vee c(x) = c(x)$ | [118 → 170] |
| 232 | $x \vee y = y \vee x$ | [34 → 32] |
| 239 | $(x \vee y) \vee y = x \vee y$ | [127 → 232] |
| 245 | $x \vee (y \vee z) = (x \vee z) \vee y$ | [4 → 232] |
| 268,267 | $(x \vee y) \vee x = y \vee x$ | [232 → 239] |
| 354,353 | $c(x \vee y) \vee c(y \vee x) = c(x \vee y)$ | [267 → 153] |
| 411 | $c(x \vee y) = c(y \vee x)$ | [267 → 203,354] |
| 642,641 | $(x \vee y) \vee z = x \vee (y \vee z)$ | [232 → 245] |
| 646 | $x \vee (y \vee z) = x \vee (z \vee y)$ | [267 → 245,642,642,268,268,642] |
| 1231 | $x \vee (y \vee c(x)) = c(x) \vee x$ | [143 → 32] |
| 1243 | $c(x) \vee x = x \vee (y \vee c(x))$ | [1231] |
| 1255,1254 | $c(x \vee y) \vee (x \vee y) = x \vee (y \vee c(x))$ | [153 → 1231,642] |
| 1264,1263 | $x \vee (y \vee (z \vee c(x \vee y))) = x \vee (y \vee c(x))$ | [641 → 1231,1255] |
| 1472,1471 | $c(x \vee y) \vee (y \vee x) = y \vee (x \vee c(y))$ | [411 → 1243,642,1264] |
| 1474 | $x \vee (y \vee c(x)) = y \vee (x \vee c(y))$ | [646 → 1243,1472,642,1264] |
| 1482,1481 | $x \vee (y \vee c(x)) = x \vee c(x)$ | [232 → 1243] |
| 1485 | $x \vee c(x) = y \vee c(y)$ | [1474,1482,1482] |
| 1486 | $\$Ans(ONE)$ | [1485,2] |

The following Mace2 jobs show that the $\mathcal{OML}$ 4-basis {AJ, B1, DM, OM} is independent.

**Countermodel OMLa**

```
x v (x ^ y) = x.                % B1
x ^ y = c(c(x) v c(y)).         % DM
```

```
x v (c(x) ^ (x v y)) = x v y.     % OM
A v (B v C) != B v (A v C).       % denial of AJ
```

C: 0 B: 0 A: 1

| c: | 0 | 1 |
|---|---|---|
| | 0 | 0 |

| ∧: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |

| ∨: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |

Table 8: OMLa.out

**Countermodel OMLb**

```
x v (y v z) = y v (x v z).        % AJ
x ^ y = c(c(x) v c(y)).           % DM
x v (c(x) ^ (x v y)) = x v y.     % OM
A v (A ^ B) != A.                 % denial of B1
```

B: 0 A: 1

| c: | 0 | 1 |
|---|---|---|
| | 0 | 0 |

| ∨: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |

| ∧: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |

Table 9: OMLb.out

**Countermodel OMLc**

```
x v (y v z) = y v (x v z).        % AJ
x v (x ^ y) = x.                  % B1
x v (c(x) ^ (x v y)) = x v y.     % OM
A ^ B != c(c(A) v c(B)).          % denial of DM
```

B: 0 A: 1

| c: | 0 | 1 |
|---|---|---|
| | 0 | 0 |

| ∨: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

| ∧: | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 1 |

Table 10: OMLc.out

**Countermodel OMLd**

```
x v (y v z) = y v (x v z).        % AJ
x v (x ^ y) = x.                  % B1
x ^ y = c(c(x) v c(y)).           % DM
A v (c(A) ^ (A v B)) != A v B.    % denial of OM
```

B: 0 A: 1

| c: | 0 | 1 |
|---|---|---|
| | 1 | 1 |

| ∨: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

| ∧: | 0 | 1 |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 1 | 1 |

Table 11: OMLd.out

### 2.1.4 Modular Ortholattices

The following is a 6-basis for modular ortholattices ($\mathcal{MOL}$) in terms of join ($\vee$), meet ($\wedge$), and complement ($c$).

| | |
|---|---|
| $x \vee (y \vee z) = y \vee (x \vee z)$ | % AJ |
| $x \vee (x \wedge y) = x$ | % B1 |
| $x \wedge y = c(c(x) \vee c(y))$ | % DM |
| $c(c(x)) = x$ | % CC |
| $x \vee c(x) = y \vee c(y)$ | % ONE |
| $x \vee (y \wedge (x \vee z)) = x \vee (z \wedge (x \vee y))$ | % MOD |

This basis is simply our ortholattice 5-basis plus the modularity law MOD. The following Otter proof shows that the orthomodular law OM holds in $\mathcal{MOL}$.

**Proof MOL1**

| | | |
|---|---|---|
| 3 | $x \vee (x \wedge y) = x$ | [] |
| 6,5 | $x \wedge y = c(c(x) \vee c(y))$ | [] |
| 8,7 | $c(c(x)) = x$ | [] |
| 9 | $x \vee c(x) = y \vee c(y)$ | [] |
| 10 | $x \vee (y \wedge (x \vee z)) = x \vee (z \wedge (x \vee y))$ | [] |
| 11 | $x \vee c(c(y) \vee c(x \vee z)) = x \vee c(c(z) \vee c(x \vee y))$ | [10,6,6] |
| 12 | $A \vee (c(A) \wedge (A \vee B)) \neq A \vee B \mid \$Ans(OM)$ | [] |
| 13 | $A \vee c(A \vee c(A \vee B)) \neq A \vee B \mid \$Ans(OM)$ | [12,6,8] |
| 14 | $x \vee c(c(x) \vee c(y)) = x$ | [3:6] |
| 18 | $c(x) \vee x = y \vee c(y)$ | [7 → 9] |
| 19 | $x \vee c(x) = c(y) \vee y$ | [18] |
| 40,39 | $x \vee c(c(y) \vee y) = x$ | [19 → 14] |
| 50 | $x \vee c(x \vee c(x \vee y)) = x \vee y$ | [19 → 11,40,8,8] |
| 52 | $\$Ans(OM)$ | [50,13] |

A more common modularity law is

$$x \vee (y \wedge (x \vee z)) = (x \vee y) \wedge (x \vee z) \quad \text{\% MOD2}$$

The following are Otter proofs that MOD and MOD2 are equivalent in lattice theory.

**Proof ML1**

| | | |
|---|---|---|
| 2 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [] |
| 3 | $x \wedge (y \wedge z) = y \wedge (x \wedge z)$ | [] |
| 4 | $x \vee (x \wedge y) = x$ | [] |
| 7,6 | $x \wedge (x \vee y) = x$ | [] |
| 8 | $x \vee (y \wedge (x \vee z)) = x \vee (z \wedge (x \vee y))$ | [] |
| 9 | $A \vee (B \wedge (A \vee C)) \neq (A \vee B) \wedge (A \vee C) \mid \$Ans(MOD2)$ | [] |
| 10 | $(A \vee B) \wedge (A \vee C) \neq A \vee (B \wedge (A \vee C)) \mid \$Ans(MOD2)$ | [9] |
| 13 | $x \vee (y \vee (x \wedge z)) = y \vee x$ | [4 → 2] |
| 15 | $x \wedge x = x$ | [4 → 6] |
| 17 | $x \wedge (y \vee (x \vee z)) = x$ | [2 → 6] |
| 20,19 | $x \vee x = x$ | [6 → 4] |
| 21 | $x \wedge (y \wedge x) = y \wedge x$ | [15 → 3] |
| 23 | $x \wedge (y \wedge (x \vee z)) = y \wedge x$ | [6 → 3] |

| | | |
|---|---|---|
| 31 | $x \vee ((x \vee y) \vee y) = x \vee y$ | [2 → 19] |
| 36,35 | $x \wedge (y \vee x) = x$ | [19 → 17] |
| 47 | $x \vee (y \wedge x) = x$ | [19 → 8,7,20] |
| 59 | $(x \wedge (y \vee z)) \wedge (y \vee (z \wedge (y \vee x))) = x \wedge (y \vee z)$ | [8 → 35] |
| 64,63 | $(x \vee y) \vee y = x \vee y$ | [35 → 47] |
| 71 | $x \vee (x \vee y) = x \vee y$ | [31:64] |
| 81 | $(x \vee y) \wedge y = y$ | [35 → 21,36] |
| 302 | $x \vee (y \vee (z \wedge (y \vee x))) = y \vee x$ | [8 → 13] |
| 311,310 | $(x \vee y) \wedge (x \vee (y \wedge z)) = x \vee (y \wedge z)$ | [13 → 81] |
| 356 | $x \wedge y = y \wedge x$ | [35 → 23] |
| 367 | $x \wedge (y \wedge z) = (x \wedge z) \wedge y$ | [3 → 356] |
| 369 | $(A \vee C) \wedge (A \vee B) \neq A \vee (B \wedge (A \vee C)) \mid \$Ans(MOD2)$ | [356 → 10] |
| 2164 | $((x \vee y) \wedge (x \vee z)) \wedge (x \vee (z \wedge (x \vee y))) = (x \vee y) \wedge (x \vee z)$ | [71 → 59] |
| 2222,2221 | $(x \wedge y) \wedge z = x \wedge (y \wedge z)$ | [356 → 367] |
| 2260,2259 | $(x \vee y) \wedge (x \vee (z \wedge (x \vee y))) = (x \vee y) \wedge (x \vee z)$ | [2164:2222,311] |
| 4218 | $(x \vee y) \wedge (x \vee z) = x \vee (z \wedge (x \vee y))$ | [302 → 81,2260] |
| 4219 | $\$Ans(MOD2)$ | [4218,369] |

**Proof ML2**

| | | |
|---|---|---|
| 2 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [] |
| 3 | $x \wedge (y \wedge z) = y \wedge (x \wedge z)$ | [] |
| 4 | $x \vee (x \wedge y) = x$ | [] |
| 6 | $x \wedge (x \vee y) = x$ | [] |
| 8 | $x \vee (y \wedge (x \vee z)) = (x \vee y) \wedge (x \vee z)$ | [] |
| 10,9 | $(x \vee y) \wedge (x \vee z) = x \vee (y \wedge (x \vee z))$ | [8] |
| 11 | $A \vee (B \wedge (A \vee C)) \neq A \vee (C \wedge (A \vee B)) \mid \$Ans(MOD)$ | [] |
| 12 | $A \vee (C \wedge (A \vee B)) \neq A \vee (B \wedge (A \vee C)) \mid \$Ans(MOD)$ | [11] |
| 19 | $x \wedge (y \vee (x \vee z)) = x$ | [2 → 6] |
| 21 | $x \vee x = x$ | [6 → 4] |
| 25 | $x \wedge (y \wedge (x \vee z)) = y \wedge x$ | [6 → 3] |
| 37 | $x \wedge (y \vee x) = x$ | [21 → 19] |
| 363 | $x \wedge y = y \wedge x$ | [37 → 25] |
| 375 | $x \vee (y \wedge (x \vee z)) = x \vee (z \wedge (x \vee y))$ | [9 → 363,10] |
| 376 | $\$Ans(MOD)$ | [375,12] |

The following Mace2 jobs show that the $\mathcal{MOL}$ 6-basis {AJ, B1, DM, CC, ONE, MOD} is independent.

**Countermodel MOLa**

```
x v (x ^ y) = x.                        % B1
x ^ y = c(c(x) v c(y)).                 % DM
c(c(x)) = x.                            % CC
x v c(x) = y v c(y).                    % ONE
x v (y ^ (x v z)) = x v (z ^ (x v y)).  % MOD
A v (B v C) != B v (A v C).             % denial of AJ
```

C: 0 B: 0 A: 1

| c: | 0 | 1 | 2 |
|---|---|---|---|
| | 0 | 2 | 1 |

| ∧: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 |
| 2 | 2 | 0 | 2 |

| ∨: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 |
| 2 | 2 | 0 | 2 |

Table 12: MOLa.out

**Countermodel MOLb**

```
x v (y v z) = y v (x v z).                % AJ
x ^ y = c(c(x) v c(y)).                   % DM
c(c(x)) = x.                              % CC
x v c(x) = y v c(y).                      % ONE
x v (y ^ (x v z)) = x v (z ^ (x v y)).    % MOD
A v (A ^ B) != A.                         % denial of B1
```

B: 0 A: 1

| c: | 0 | 1 |
|---|---|---|
| | 0 | 1 |

| ∨: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |

| ∧: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |

Table 13: MOLb.out

**Countermodel MOLc**

```
x v (y v z) = y v (x v z).                % AJ
x v (x ^ y) = x.                          % B1
c(c(x)) = x.                              % CC
x v c(x) = y v c(y).                      % ONE
x v (y ^ (x v z)) = x v (z ^ (x v y)).    % MOD
A ^ B != c(c(A) v c(B)).                  % denial of DM
```

B: 0 A: 0

| c: | 0 | 1 |
|---|---|---|
| | 1 | 0 |

| ∨: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

| ∧: | 0 | 1 |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 1 | 1 |

Table 14: MOLc.out

**Countermodel MOLd**

```
x v (y v z) = y v (x v z).                % AJ
x v (x ^ y) = x.                          % B1
x ^ y = c(c(x) v c(y)).                   % DM
x v c(x) = y v c(y).                      % ONE
x v (y ^ (x v z)) = x v (z ^ (x v y)).    % MOD
c(c(A)) != A.                             % denial of CC
```

A: 0

| c: | 0 | 1 | 2 |
|---|---|---|---|
| | 1 | 2 | 1 |

| ∨: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 |
| 2 | 0 | 1 | 2 |

| ∧: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 2 | 2 | 2 |
| 1 | 2 | 1 | 2 |
| 2 | 2 | 2 | 2 |

Table 15: MOLd.out

**Countermodel MOLe**

```
x v (y v z) = y v (x v z).                % AJ
x v (x ^ y) = x.                          % B1
x ^ y = c(c(x) v c(y)).                   % DM
c(c(x)) = x.                              % CC
x v (y ^ (x v z)) = x v (z ^ (x v y)).    % MOD
A v c(A) != B v c(B).                     % denial of ONE
```

B: 0 A: 1

| c: | 0 | 1 | 2 |
|---|---|---|---|
| | 2 | 1 | 0 |

| ∨: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 2 | 0 | 1 | 2 |

| ∧: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 1 | 2 |
| 1 | 1 | 1 | 2 |
| 2 | 2 | 2 | 2 |

Table 16: MOLe.out

**Countermodel MOLf**

```
x v (y v z) = y v (x v z).                % AJ
x v (x ^ y) = x.                          % B1
x ^ y = c(c(x) v c(y)).                   % DM
c(c(x)) = x.                              % CC
x v c(x) = y v c(y).                      % ONE
x v c(x) = 1.                             % ONEa
c(1) = 0.                                 % DEF_0
A v (B ^ (A v C)) != A v (C ^ (A v B)).   % denial of MOD
```

C: 2 B: 3 A: 4

| c: | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| | 1 | 0 | 3 | 2 | 5 | 4 |

| ∨: | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 1 | 2 | 1 | 1 | 5 |
| 3 | 3 | 1 | 1 | 3 | 3 | 1 |
| 4 | 4 | 1 | 1 | 3 | 4 | 1 |
| 5 | 5 | 1 | 5 | 1 | 1 | 5 |

| ∧: | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 | 3 | 4 | 5 |
| 2 | 0 | 2 | 2 | 0 | 0 | 2 |
| 3 | 0 | 3 | 0 | 3 | 4 | 0 |
| 4 | 0 | 4 | 0 | 4 | 4 | 0 |
| 5 | 0 | 5 | 2 | 0 | 0 | 5 |

Table 17: MOLf.out

### 2.1.5 Boolean Algebra

The following is a 4-basis for Boolean algebra ($\mathcal{BA}$) in terms of join ($\vee$), meet ($\wedge$), and complement ($c$).

$x \vee (y \vee z) = y \vee (x \vee z)$ % AJ
$x \wedge y = c(c(x) \vee c(y))$ % DM
$x \vee c(x) = y \vee c(y)$ % ONE
$(x \vee c(y)) \wedge (x \vee y) = x$ % CUT

The following are proofs of distributivity, modularity, CC, and B1 from the $\mathcal{BA}$ 4-basis.

**Proof BA1**

| | | |
|---|---|---|
| 2 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [] |
| 3 | $x \wedge y = c(c(x) \vee c(y))$ | [] |
| 4 | $c(c(x) \vee c(y)) = x \wedge y$ | [3] |
| 7 | $(x \vee c(y)) \wedge (x \vee y) = x$ | [] |
| 10,9 | $x \vee c(x) = 1$ | [] |
| 12,11 | $c(1) = 0$ | [] |
| 13 | $A \wedge (B \vee C) \neq (A \wedge B) \vee (A \wedge C) \ \mid \ \$Ans(DIST1)$ | [] |
| 14 | $x \vee (y \vee (z \vee u)) = z \vee (x \vee (y \vee u))$ | [2 → 2] |
| 15 | $x \vee (y \vee (z \vee u)) = y \vee (z \vee (x \vee u))$ | [14] |
| 17,16 | $1 \vee 0 = 1$ | [11 → 9] |
| 18 | $x \vee (y \vee c(x)) = y \vee 1$ | [9 → 2] |
| 20 | $x \vee 1 = 1 \vee (x \vee 0)$ | [16 → 2] |
| 21 | $1 \vee (x \vee 0) = x \vee 1$ | [20] |
| 22 | $c(0 \vee c(x)) = 1 \wedge x$ | [11 → 4] |
| 24 | $c((x \wedge y) \vee c(z)) = (c(x) \vee c(y)) \wedge z$ | [4 → 4] |
| 27,26 | $c(c(x) \vee 0) = x \wedge 1$ | [11 → 4] |
| 29,28 | $c(c(x) \vee (y \wedge z)) = x \wedge (c(y) \vee c(z))$ | [4 → 4] |
| 30 | $x \wedge c(x) = 0$ | [9 → 4,12] |
| 32 | $(c(x) \vee c(y)) \vee (x \wedge y) = 1$ | [4 → 9] |
| 34 | $1 \wedge 0 = 0$ | [11 → 30] |
| 36 | $(c(x) \vee c(y)) \wedge (x \wedge y) = 0$ | [4 → 30] |
| 38 | $x \vee (1 \vee (y \vee 0)) = y \vee (x \vee 1)$ | [20 → 2] |
| 39 | $x \vee (y \vee 1) = y \vee (1 \vee (x \vee 0))$ | [38] |
| 41,40 | $(x \vee 0) \wedge (x \vee 1) = x$ | [11 → 7] |
| 42 | $(x \vee (y \wedge z)) \wedge (x \vee (c(y) \vee c(z))) = x$ | [4 → 7] |
| 45,44 | $1 \wedge (x \vee x) = x$ | [9 → 7] |
| 46 | $(x \vee 0) \wedge (1 \vee (x \vee 0)) = x$ | [20 → 7,12] |
| 48 | $(1 \vee c(0)) \wedge 1 = 1$ | [16 → 7] |
| 50 | $(x \vee c(c(x))) \wedge 1 = x$ | [9 → 7] |
| 52 | $(x \vee c(y \vee z)) \wedge (y \vee (x \vee z)) = x$ | [2 → 7] |
| 55,54 | $1 \wedge (x \vee ((x \vee y) \vee y)) = x \vee y$ | [2 → 44] |
| 58 | $x \vee (y \vee 1) = 1 \vee (x \vee (y \vee 0))$ | [21 → 2] |
| 59 | $1 \vee (x \vee (y \vee 0)) = x \vee (y \vee 1)$ | [58] |
| 63,62 | $c(0 \vee 0) = 1 \wedge 1$ | [11 → 22] |
| 64 | $c(0 \vee (x \wedge y)) = 1 \wedge (c(x) \vee c(y))$ | [4 → 22] |
| 66 | $(x \vee (1 \wedge y)) \wedge (x \vee (0 \vee c(y))) = x$ | [22 → 7] |
| 70 | $(0 \vee c(x)) \vee (1 \wedge x) = 1$ | [22 → 9] |
| 72 | $(x \vee (1 \wedge 1)) \wedge (x \vee (0 \vee 0)) = x$ | [62 → 7] |
| 85,84 | $c((x \wedge y) \vee 0) = (c(x) \vee c(y)) \wedge 1$ | [4 → 26] |
| 90 | $(c(x) \vee 0) \vee (x \wedge 1) = 1$ | [26 → 9] |
| 94 | $x \vee (y \vee (z \vee c(x))) = y \vee (z \vee 1)$ | [9 → 14] |
| 98 | $x \vee (y \vee (z \vee u)) = z \vee (y \vee (x \vee u))$ | [2 → 14] |
| 105 | $(x \vee c(y \vee (z \vee u))) \wedge (z \vee (x \vee (y \vee u))) = x$ | [14 → 7] |
| 127 | $1 \wedge (x \vee (y \vee ((x \vee (y \vee z)) \vee z))) = x \vee (y \vee z)$ | [15 → 44] |
| 130 | $(x \vee c(y \vee (z \vee u))) \wedge (y \vee (z \vee (x \vee u))) = x$ | [15 → 7] |
| 137 | $(c(x) \vee 0) \vee (y \vee (x \wedge 1)) = y \vee 1$ | [26 → 18] |
| 141 | $(c(x) \vee c(y)) \vee (z \vee (x \wedge y)) = z \vee 1$ | [4 → 18] |
| 161,160 | $c(x \vee c(y)) = (c(x \vee 0) \vee c(x \vee 1)) \wedge y$ | [40 → 24] |
| 163,162 | $((1 \wedge 1) \vee c(0 \vee 1)) \wedge x = 1 \wedge x$ | [34 → 24,161,63,12,10] |
| 174 | $(0 \vee (1 \wedge 1)) \vee 0 = 1$ | [62 → 70,45] |
| 178 | $(0 \vee c(x \vee x)) \vee x = 1$ | [44 → 70] |
| 182 | $1 \wedge ((0 \vee (1 \wedge 1)) \vee 1) = 0 \vee (1 \wedge 1)$ | [174 → 40] |

| | | |
|---|---|---|
| 185,184 | $(0 \vee (1 \wedge 1)) \vee 1 = 1 \vee 1$ | [174 → 21] |
| 191 | $0 \vee (1 \wedge 1) = 1$ | [182:185,45] |
| 196,195 | $c((x \wedge 1) \vee (y \wedge z)) = (c(x) \vee 0) \wedge (c(y) \vee c(z))$ | [26 → 28] |
| 197 | $c(c(x) \vee 1) = x \wedge (((0 \vee c(1 \vee 1)) \wedge 0) \vee 0)$ | [48 → 28,161,17,12,12] |
| 199 | $c(c(x) \vee y) = x \wedge (0 \vee c(y \vee y))$ | [44 → 28,12] |
| 200 | $c(c(x) \vee y) = x \wedge (c(y \vee 0) \vee c(y \vee 1))$ | [40 → 28] |
| 201 | $x \wedge (0 \vee c(y \vee y)) = c(c(x) \vee y)$ | [199] |
| 202 | $x \wedge (c(y \vee 0) \vee c(y \vee 1)) = c(c(x) \vee y)$ | [200] |
| 205 | $(c(x) \vee (y \wedge z)) \vee (x \wedge (c(y) \vee c(z))) = 1$ | [28 → 9] |
| 210 | $(0 \vee c(1 \wedge 1)) \wedge 1 = 0$ | [191 → 7] |
| 212 | $0 \vee (x \vee (1 \wedge 1)) = x \vee 1$ | [191 → 2] |
| 226 | $(((c(x \vee 0) \vee c(x \vee 1)) \wedge c(x)) \vee 0) \vee x = 1$ | [50 → 90,161] |
| 234 | $((x \wedge 1) \vee c(y)) \vee ((c(x) \vee 0) \wedge y) = 1$ | [26 → 32] |
| 240 | $(c(x \vee 0) \vee c(x \vee 1)) \vee x = 1$ | [40 → 32] |
| 250 | $x \vee ((0 \vee c((x \vee y) \vee (x \vee y))) \vee y) = 1$ | [2 → 178] |
| 255 | $x \vee (y \vee 1) = (0 \vee c(z \vee z)) \vee (x \vee (y \vee z))$ | [178 → 14] |
| 258 | $x \vee 1 = (0 \vee c(y \vee y)) \vee (x \vee y)$ | [178 → 2] |
| 260 | $(0 \vee c(x \vee x)) \vee (y \vee (z \vee x)) = y \vee (z \vee 1)$ | [255] |
| 263,262 | $((1 \wedge (1 \wedge 1)) \vee 0) \vee 0 = 1$ | [210 → 90,161,63,163] |
| 278 | $c(c(x) \vee y) = x \wedge (c(y \vee 0) \vee c(1 \vee (y \vee 0)))$ | [46 → 28] |
| 287,286 | $(0 \vee 0) \vee 1 = 0 \vee 1$ | [90 → 212,12] |
| 306 | $(x \vee c(1 \vee (y \vee 0))) \wedge (y \vee (x \vee 1)) = x$ | [38 → 7] |
| 312,311 | $(1 \wedge (1 \wedge 1)) \vee 0 = 1$ | [262 → 46,263,45] |
| 314 | $1 \wedge (1 \wedge 1) = 1$ | [311 → 46,312,45] |
| 322 | $x \vee (y \vee 1) = x \vee (1 \vee (y \vee 0))$ | [2 → 39] |
| 325 | $x \vee (1 \vee (y \vee 0)) = x \vee (y \vee 1)$ | [322] |
| 334 | $(x \vee c(y \vee 1)) \wedge (y \vee (1 \vee (x \vee 0))) = x$ | [39 → 7] |
| 376 | $(x \vee y) \wedge (x \vee (0 \vee c(y \vee y))) = x$ | [44 → 42,12] |
| 380 | $1 \wedge (0 \vee (0 \vee 0)) = 0$ | [191 → 42,12,12] |
| 388 | $(x \vee (y \wedge (c(z) \vee 0))) \wedge (x \vee (c(y) \vee (z \wedge 1))) = x$ | [26 → 42] |
| 390 | $(x \vee (y \wedge x)) \wedge (c(y) \vee 1) = x$ | [18 → 42] |
| 392 | $(x \vee (y \wedge z)) \wedge (c(y) \vee (x \vee c(z))) = x$ | [2 → 42] |
| 399,398 | $(0 \vee c(0 \vee (0 \vee 0))) \vee 0 = 1$ | [380 → 70] |
| 422 | $(x \vee (1 \wedge 1)) \wedge (0 \vee (x \vee 0)) = x$ | [62 → 52] |
| 424 | $(x \vee (y \wedge 1)) \wedge (c(y) \vee (x \vee 0)) = x$ | [26 → 52] |
| 440 | $(x \vee c((0 \vee c((x \vee y) \vee (x \vee y))) \vee y)) \wedge 1 = x$ | [178 → 52] |
| 444 | $(1 \vee c(x \vee (y \vee 0))) \wedge (y \vee (x \vee 1)) = 1$ | [38 → 52] |
| 448 | $(x \vee c(y \vee (z \vee u))) \wedge (z \vee (y \vee (x \vee u))) = x$ | [14 → 52] |
| 466 | $((x \vee x) \vee x) \wedge (0 \vee 1) = x \vee x$ | [44 → 390,12] |
| 500 | $1 \wedge (x \vee (y \vee ((y \vee (x \vee z)) \vee z))) = y \vee (x \vee z)$ | [14 → 54] |
| 505 | $0 \vee c(0 \vee (0 \vee 0)) = 1$ | [398 → 46,399,45] |
| 520 | $0 \vee (x \vee c(0 \vee (0 \vee 0))) = x \vee 1$ | [505 → 2] |
| 561,560 | $(0 \vee c((x \vee 0) \vee (x \vee 0))) \vee (x \vee 1) = 1 \vee 1$ | [178 → 59] |
| 580 | $1 \wedge (c(x \vee 0) \vee c(1 \vee (x \vee 0))) = c(0 \vee x)$ | [46 → 64] |
| 582 | $c(0 \vee x) = 1 \wedge (0 \vee c(x \vee x))$ | [44 → 64,12] |
| 584 | $1 \wedge (0 \vee c(x \vee x)) = c(0 \vee x)$ | [582] |
| 618 | $(x \vee (1 \wedge y)) \wedge (0 \vee (x \vee c(y))) = x$ | [2 → 66] |
| 631,630 | $((1 \wedge 1) \vee c(0 \vee 1)) \vee 0 = 1$ | [62 → 240] |
| 643 | $(1 \wedge 1) \vee c(0 \vee 1) = 1$ | [240 → 46,63,631,45,63] |
| 658 | $c(x \vee 0) = (0 \vee c(x \vee x)) \wedge 1$ | [44 → 84,12] |
| 662 | $(0 \vee c(x \vee x)) \wedge 1 = c(x \vee 0)$ | [658] |
| 665 | $(x \vee ((c(y) \vee c(z)) \wedge 1)) \wedge ((y \wedge z) \vee (x \vee 0)) = x$ | [84 → 52] |
| 669 | $((x \wedge y) \vee 0) \wedge ((c(x) \vee c(y)) \wedge 1) = 0$ | [84 → 30] |
| 683 | $(1 \wedge 1) \vee c(1 \vee (0 \vee 0)) = 1$ | [20 → 643] |

| | | |
|---|---|---|
| 685 | $1 \wedge (0 \vee ((1 \wedge 1) \vee 1)) = 1 \wedge 1$ | [643 → 52] |
| 688,687 | $(1 \wedge 1) \vee 1 = (0 \vee 1) \vee 1$ | [643 → 18] |
| 703,702 | $1 \wedge 1 = 0 \vee 1$ | [685:688,55] |
| 710 | $(0 \vee 1) \vee c(1 \vee (0 \vee 0)) = 1$ | [683:703] |
| 732 | $(x \vee (0 \vee 1)) \wedge (0 \vee (x \vee 0)) = x$ | [422:703] |
| 755,754 | $1 \wedge (0 \vee 1) = 1$ | [314:703] |
| 772 | $0 \vee (x \vee (0 \vee 1)) = x \vee 1$ | [212:703] |
| 778 | $0 \vee (0 \vee 1) = 1$ | [191:703] |
| 788 | $(x \vee (0 \vee 1)) \wedge (x \vee (0 \vee 0)) = x$ | [72:703] |
| 791,790 | $c(0 \vee 0) = 0 \vee 1$ | [62:703] |
| 792 | $(1 \vee (0 \vee 1)) \wedge (0 \vee 1) = 1$ | [702 → 390,12] |
| 794 | $c(c(x) \vee (0 \vee 1)) = x \wedge (0 \vee 0)$ | [702 → 28,12,12] |
| 806 | $0 \vee (1 \vee (0 \vee 0)) = 1$ | [20 → 778] |
| 811,810 | $0 \vee (0 \vee (x \vee 1)) = x \vee 1$ | [778 → 15] |
| 814 | $c((0 \vee 1) \vee (x \wedge y)) = (0 \vee 0) \wedge (c(x) \vee c(y))$ | [790 → 28] |
| 840 | $0 \vee (1 \vee (x \vee (0 \vee 0))) = x \vee 1$ | [806 → 15] |
| 860 | $0 \vee (0 \vee (1 \vee (x \vee 0))) = x \vee 1$ | [39 → 772] |
| 862 | $x \vee 1 = 0 \vee (0 \vee (1 \vee (x \vee 0)))$ | [860] |
| 866,865 | $(0 \vee (1 \vee 1)) \wedge (0 \vee 1) = 1$ | [39 → 792,17] |
| 925,924 | $(1 \vee (0 \vee 0)) \vee 1 = (0 \vee 1) \vee 1$ | [710 → 18] |
| 1073,1072 | $1 \wedge (0 \vee c((x \vee y) \vee (x \vee y))) = c(x \vee (0 \vee y))$ | [2 → 582] |
| 1187 | $(0 \vee (x \vee 1)) \wedge (0 \vee (x \vee 0)) = x$ | [2 → 732] |
| 1211 | $(x \vee c(y \vee (z \vee c(x)))) \wedge (z \vee (y \vee 1)) = x$ | [18 → 105] |
| 1229 | $(0 \vee (x \vee 1)) \wedge (x \vee (0 \vee 0)) = x$ | [2 → 788] |
| 1232,1231 | $0 \vee c((0 \vee 0) \vee (0 \vee 0)) = (1 \vee 1) \wedge 1$ | [178 → 788,561] |
| 1294,1293 | $(c((0 \vee 0) \vee 0) \vee c(0 \vee 1)) \vee 1 = 0 \vee (1 \vee 1)$ | [240 → 840,287] |
| 1296,1295 | $((1 \vee 1) \wedge 1) \vee 1 = 0 \vee (1 \vee 1)$ | [178 → 840,1232] |
| 1318 | $c(x \vee 1) = c(0 \vee (0 \vee (1 \vee (x \vee 0))))$ | [860 → 582,1073] |
| 1325,1324 | $0 \vee (0 \vee (1 \vee (x \vee 0))) = 1 \vee (x \vee 0)$ | [20 → 862] |
| 1327 | $c(x \vee 1) = c(1 \vee (x \vee 0))$ | [1318:1325] |
| 1440 | $c((0 \vee 0) \vee 0) \vee c(0 \vee 1) = (1 \vee 1) \wedge 1$ | [240 → 1229,287,1294,811,287] |
| 1589,1588 | $c(1 \vee (c(x) \vee 0)) = x \wedge (0 \vee c(1 \vee 1))$ | [20 → 199] |
| 1624 | $c((1 \wedge c(x \vee x)) \vee x) = 0$ | [7 → 201,161,791,10] |
| 1683 | $((1 \wedge c(x \vee x)) \vee x) \vee 0 = 1$ | [1624 → 9] |
| 1704,1703 | $x \vee (((1 \wedge c(y \vee y)) \vee y) \vee 1) = x \vee (1 \vee 1)$ | [1683 → 325] |
| 1705 | $(1 \wedge c(x \vee x)) \vee x = 1$ | [1683 → 1187,1704,866] |
| 1755 | $x \vee 1 = (1 \wedge c(y \vee y)) \vee (x \vee y)$ | [1705 → 2] |
| 1762 | $(1 \wedge c(x \vee x)) \vee (y \vee x) = y \vee 1$ | [1755] |
| 1867 | $(x \vee y) \wedge (0 \vee (x \vee c(y \vee y))) = x$ | [2 → 376] |
| 1877 | $(x \vee ((c(y) \vee 0) \wedge 1)) \wedge ((y \wedge 1) \vee (x \vee 0)) = x$ | [26 → 424] |
| 1911 | $(0 \vee (0 \vee 0)) \vee (x \vee 1) = 0 \vee (x \vee 1)$ | [520 → 94] |
| 2367 | $(c(x) \vee (1 \wedge y)) \vee (x \wedge (0 \vee c(y))) = 1$ | [11 → 205] |
| 2369 | $(0 \vee (x \wedge x)) \vee c(x) = 1$ | [44 → 205,12] |
| 2378,2377 | $(0 \vee (x \wedge x)) \vee 1 = x \vee 1$ | [2369 → 18] |
| 2396 | $(0 \vee (x \wedge x)) \vee (y \vee c(x)) = y \vee 1$ | [2369 → 2] |
| 2403,2402 | $1 \vee ((0 \vee (x \wedge x)) \vee 0) = x \vee 1$ | [20 → 2377] |
| 2462,2461 | $x \vee (((1 \wedge c(0)) \vee 0) \vee 1) = x \vee (1 \vee 1)$ | [226 → 325,791,10] |
| 2463 | $(1 \wedge c(0)) \vee 0 = 1$ | [226 → 1187,791,10,2462,866,791,10] |
| 2468,2467 | $(1 \wedge c(0)) \vee 1 = 1 \vee 1$ | [2463 → 1762,791,755] |
| 2479 | $1 \wedge c(0) = 1$ | [2463 → 1187,2468,866] |
| 2484,2483 | $(c(0) \vee 1) \wedge (0 \vee 1) = c(0)$ | [2479 → 390,12] |
| 2485 | $(x \vee 1) \wedge (0 \vee (x \vee c(c(0)))) = x$ | [2479 → 618] |
| 2532 | $(1 \vee (c(0) \vee 0)) \wedge (0 \vee 1) = c(0)$ | [20 → 2483] |
| 2564 | $0 \vee (c(0) \wedge c(0)) = c(0)$ | [2369 → 2485,2378,2484] |

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|---|---|---|
| 2583 | $(0 \vee c(c(0) \wedge c(0))) \wedge c(0) = 0$ | [2564 → 7] |
| 2605 | $((0 \wedge (0 \vee c(1 \vee 1))) \vee c(0 \vee 1)) \wedge 1 = 0 \wedge 1$ | [2532 → 84,27,1589] |
| 2613 | $((0 \wedge (0 \vee c(1 \vee 1))) \vee c(0 \vee 1)) \vee c(0) = 1$ | [2532 → 32,1589] |
| 3123 | $(x \vee ((c(y) \vee 0) \wedge z)) \wedge ((y \wedge 1) \vee (x \vee c(z))) = x$ | [26 → 392] |
| 3344 | $0 \vee ((x \vee 1) \wedge (x \vee 1)) = x \vee 1$ | [2369 → 334,2403,55] |
| 3353,3352 | $(c(x \vee 1) \vee c(x \vee 1)) \vee (x \vee 1) = 0 \vee 1$ | [3344 → 141] |
| 3541 | $(0 \vee (0 \vee 0)) \vee (0 \vee (1 \vee 1)) = 1 \vee 1$ | [1295 → 1911,1296,811] |
| 3895 | $((1 \wedge c(x)) \vee (1 \wedge x)) \vee 0 = 1$ | [7 → 2367,161,791,10] |
| 3924,3923 | $((1 \wedge c(x)) \vee (1 \wedge x)) \vee 1 = 1 \vee 1$ | [3895 → 1762,791,755] |
| 3928 | $(1 \wedge c(x)) \vee (1 \wedge x) = 1$ | [3895 → 1187,3924,866] |
| 3964 | $(1 \wedge x) \vee c(0 \vee x) = 1$ | [584 → 3928,161,791,10,45] |
| 4033 | $x \vee c(0 \vee (x \vee x)) = 1$ | [44 → 3964] |
| 4112,4111 | $(1 \wedge x) \vee 1 = (0 \vee x) \vee 1$ | [3964 → 2396,2378] |
| 4204 | $x \vee c(x \vee (0 \vee x)) = 1$ | [2 → 4033] |
| 4218,4217 | $1 \wedge (0 \vee (x \vee (x \vee x))) = x$ | [4033 → 130] |
| 4219 | $1 \wedge (x \vee (x \vee (0 \vee x))) = x$ | [4033 → 105] |
| 4228 | $(0 \vee (x \vee x)) \vee 1 = x \vee 1$ | [4033 → 2396,2378] |
| 4300,4299 | $(x \vee (0 \vee x)) \vee 1 = x \vee 1$ | [4204 → 2396,2378] |
| 4437,4436 | $1 \wedge (c(0) \vee (c(0) \vee 1)) = c(0)$ | [94 → 4217] |
| 4474 | $1 \wedge ((1 \vee (0 \vee 0)) \vee ((0 \vee 1) \vee 1)) = 1 \vee (0 \vee 0)$ | [806 → 4219,925] |
| 4554 | $(c(0) \vee 1) \vee 1 = c(0) \vee 1$ | [18 → 4228] |
| 4567,4566 | $0 \vee (1 \vee 1) = 1$ | [4228 → 127,4218] |
| 4590,4589 | $1 \vee 1 = 0 \vee 1$ | [3541:4567,4300] |
| 4878 | $((0 \wedge (0 \vee c(0 \vee 1))) \vee c(0 \vee 1)) \vee c(0) = 1$ | [2613:4590] |
| 4884 | $((0 \wedge (0 \vee c(0 \vee 1))) \vee c(0 \vee 1)) \wedge 1 = 0 \wedge 1$ | [2605:4590] |
| 4945,4944 | $c(1 \vee (c(x) \vee 0)) = x \wedge (0 \vee c(0 \vee 1))$ | [1588:4590] |
| 4948 | $c((0 \vee 0) \vee 0) \vee c(0 \vee 1) = (0 \vee 1) \wedge 1$ | [1440:4590] |
| 4976 | $c(c(x) \vee 1) = x \wedge (((0 \vee c(0 \vee 1)) \wedge 0) \vee 0)$ | [197:4590] |
| 4982 | $(0 \vee c(x \vee x)) \vee (1 \vee x) = 0 \vee 1$ | [258 → 4589] |
| 5310,5309 | $c(0) \vee 1 = c(0)$ | [4554 → 54,4437] |
| 5347 | $c(c(0)) = 0 \wedge (0 \vee c(0 \vee 1))$ | [4554 → 1327,5310,5310,4945] |
| 5358,5357 | $c(0) = 0 \vee 1$ | [4554 → 810,5310,10,5310,5310] |
| 5406,5405 | $0 \wedge (0 \vee c(0 \vee 1)) = c(0 \vee 1)$ | [5347:5358] |
| 5424,5423 | $0 \vee 1 = 1$ | [4878:5406,5358,3353] |
| 5440,5439 | $(0 \vee 0) \wedge 1 = 0$ | [2583:5358,5424,5358,5424,703,5424,12,5358,5424] |
| 5448 | $((0 \wedge (0 \vee 0)) \vee 0) \wedge 1 = 0 \wedge 1$ | [4884:5424,12,5424,12] |
| 5456,5455 | $1 \vee (0 \vee 0) = 1$ | [4474:5424,4590,5424,925,5424,4590,5424,703,5424] |
| 5466,5465 | $0 \wedge (0 \vee 0) = 0$ | [5405:5424,12,5424,12] |
| 5476,5475 | $c(0) = 1$ | [5357:5424] |
| 5499 | $(0 \vee c(x \vee x)) \vee (1 \vee x) = 1$ | [4982:5424] |
| 5504,5503 | $c(c(x) \vee 1) = x \wedge (((0 \vee 0) \wedge 0) \vee 0)$ | [4976:5424,12] |
| 5531 | $c((0 \vee 0) \vee 0) \vee 0 = 1$ | [4948:5424,12,5424,703,5424] |
| 5535 | $c(1 \vee (c(x) \vee 0)) = x \wedge (0 \vee 0)$ | [4944:5424,12] |
| 5652,5651 | $1 \vee 1 = 1$ | [4589:5424] |
| 5838,5837 | $c(1 \vee (x \wedge y)) = (0 \vee 0) \wedge (c(x) \vee c(y))$ | [814:5424] |
| 5840,5839 | $x \wedge (((0 \vee 0) \wedge 0) \vee 0) = x \wedge (0 \vee 0)$ | [794:5424,5504] |
| 5842,5841 | $c(0 \vee 0) = 1$ | [790:5424] |
| 5862,5861 | $1 \wedge 1 = 1$ | [702:5424] |
| 5874,5873 | $((x \vee x) \vee x) \wedge 1 = x \vee x$ | [466:5424] |
| 5913,5912 | $0 \wedge 1 = 0$ | [5448:5466,5440] |
| 5948 | $c(c(x) \vee 1) = x \wedge (0 \vee 0)$ | [5503:5840] |
| 5966 | $(x \vee (0 \wedge x)) \wedge 1 = x$ | [5475 → 390,5652] |
| 6031,6030 | $(0 \vee 0) \wedge (c(x) \vee c(y)) = 0 \wedge (c(x) \vee c(y))$ | [5475 → 28,5838] |
| 6053,6052 | $c(1 \vee (x \wedge y)) = 0 \wedge (c(x) \vee c(y))$ | [5837:6031] |

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|---|---|---|
| 6257,6256 | $1 \wedge (c(x \vee (0 \wedge x)) \vee 0) = c(0 \vee x)$ | [5966 → 64,12] |
| 6383,6382 | $0 \vee 0 = 0$ | [5531 → 278,12,5842,5456,12,17,5874] |
| 6391,6390 | $c(c(x) \vee 1) = x \wedge 0$ | [5948:6383] |
| 6437,6436 | $c(1 \vee (c(x) \vee 0)) = x \wedge 0$ | [5535:6383] |
| 6499,6498 | $0 \vee (x \vee 0) = x \vee 0$ | [6382 → 2] |
| 6638 | $(1 \wedge (x \vee 0)) \vee c(x \vee 0) = 1$ | [6498 → 3964] |
| 6782 | $c(1 \vee ((x \wedge 0) \vee 0)) = (1 \vee (c(x) \vee 0)) \wedge 0$ | [6436 → 6436] |
| 6784 | $(1 \vee (x \wedge 0)) \wedge (c(x) \vee 1) = 1$ | [6436 → 444,5652] |
| 7014 | $(0 \vee (((x \wedge 1) \vee (x \wedge 0)) \wedge x)) \vee 1 = x \vee 1$ | [5499 → 94,161,27,6391,5652] |
| 7283 | $1 \wedge ((x \wedge 1) \vee (x \wedge 0)) = 1 \wedge x$ | [278 → 580,6383,5476,6383,17,12,17,6437,161,6383,5476,5424,12,17] |
| 7345 | $(1 \wedge (c(x) \vee 0)) \vee (x \wedge 1) = 1$ | [278 → 6638,6383,5476,6383,17,12,17] |
| 7391 | $(x \vee 1) \wedge (x \vee ((0 \wedge (c(y) \vee 1)) \vee (y \wedge 0))) = x$ | [6784 → 42,6053,5476,6391] |
| 7679 | $c(0 \vee x) \vee x = 1$ | [5966 → 7345,6257] |
| 7693 | $c(x \vee 0) \vee (x \vee 0) = 1$ | [6498 → 7679] |
| 7743 | $(1 \wedge x) \vee c(x) = 1$ | [160 → 7679,6383,5476,5424,12,17] |
| 7808,7807 | $(x \vee x) \wedge 1 = x$ | [7679 → 1867,161,6383,5476,5424,12,17,45,5424,161,6383,5476,5424,12,17,45] |
| 7824 | $(0 \vee c(x \vee x)) \vee 1 = c(0 \vee (y \vee x)) \vee (y \vee 1)$ | [7679 → 260] |
| 7920 | $c(x \vee 0) \vee (x \vee 1) = 1$ | [7679 → 59,5652,6499] |
| 7949,7948 | $(0 \vee x) \vee 1 = x \vee 1$ | [7679 → 2396,2378,161,6383,5476,5424,12,17,4112] |
| 8055 | $c(x \vee x) \vee 1 = c(0 \vee (y \vee x)) \vee (y \vee 1)$ | [7824:7949] |
| 8075,8074 | $(((x \wedge 1) \vee (x \wedge 0)) \wedge x) \vee 1 = x \vee 1$ | [7014:7949] |
| 8136 | $(x \vee x) \vee 1 = x \vee 1$ | [4228:7949] |
| 8141,8140 | $(1 \wedge x) \vee 1 = x \vee 1$ | [4111:7949] |
| 8152 | $(x \wedge x) \vee 1 = x \vee 1$ | [2377:7949] |
| 8207 | $(1 \wedge (c(x \vee x) \vee 0)) \vee x = 1$ | [7807 → 7345] |
| 8250 | $c(x \vee 0) = (c(x \vee x) \vee 0) \wedge 1$ | [7807 → 84,12] |
| 8260 | $(c(x \vee x) \vee 0) \wedge 1 = c(x \vee 0)$ | [8250] |
| 8288 | $x \vee c(x \vee x) = 1$ | [44 → 7743] |
| 8349 | $x \wedge (c(x) \vee c(x \vee 1)) = 0$ | [8288 → 200,12,161,27,6391,85,196,5476,41,7808,161,27,6391,8075] |
| 8353,8352 | $1 \wedge (x \vee ((x \vee y) \vee ((x \vee y) \vee y))) = x \vee y$ | [8288 → 448] |
| 8359 | $1 \wedge (x \vee (x \vee x)) = x$ | [8288 → 52] |
| 8565,8564 | $(x \vee 1) \vee (x \vee 1) = x \vee 1$ | [8136 → 500,8353] |
| 8727,8726 | $x \wedge (c(y) \vee c(y \vee 1)) = x \wedge (c(y) \vee c(y))$ | [8152 → 202,85,7808,29] |
| 8779 | $x \wedge (c(x) \vee c(x)) = 0$ | [8349:8727] |
| 8821 | $x \vee c(x \vee (x \vee x)) = 1$ | [8359 → 7743] |
| 8851 | $(c(x) \vee 1) \wedge ((x \wedge 0) \vee (x \wedge 0)) = 0$ | [6390 → 8779,6391] |
| 8877 | $(x \wedge 1) \vee (c(x) \vee 0) = 1$ | [278 → 7693,6383,5476,6383,17,12,17] |
| 8893 | $(x \vee c(c(x \vee 0) \vee (y \vee 0))) \wedge (y \vee 1) = x$ | [7693 → 448] |
| 9085 | $(x \wedge 1) \vee (c(x) \vee 1) = 1$ | [278 → 7920,6383,5476,6383,17,12,17] |
| 9089 | $c(x) \vee (x \vee 1) = 1$ | [8152 → 7920,85,7808] |
| 9110,9109 | $c(x \vee 1) \vee 1 = c(x \vee 0) \vee 1$ | [7920 → 1762,8565,8141] |
| 9199 | $c(0 \vee x) \vee (x \vee 1) = 1$ | [7948 → 9089] |
| 9221 | $x \vee (c(x) \vee 1) = 1$ | [2 → 9089] |
| 9225 | $(x \vee (x \wedge 0)) \wedge 1 = x$ | [9089 → 306,6437] |
| 9228,9227 | $c(x \vee 0) \vee 1 = c(x) \vee 1$ | [9089 → 1762,8565,8141,9110] |
| 9380,9379 | $(x \wedge 0) \vee 1 = x \vee 1$ | [9221 → 1762,8565,6391,8141] |
| 9397 | $(x \vee c(y \vee (z \vee (c(x) \vee 1)))) \wedge (y \vee (z \vee 1)) = x$ | [9221 → 130] |
| 9495 | $1 \wedge (c(x \vee (x \wedge 0)) \vee 0) = c(0 \vee x)$ | [9225 → 64,12] |
| 9516,9515 | $1 \vee ((x \wedge 0) \vee 0) = x \vee 1$ | [20 → 9379] |
| 9518,9517 | $c(x \vee 1) = (1 \vee (c(x) \vee 0)) \wedge 0$ | [6782:9516] |
| 9616,9615 | $(1 \vee (c(c(x)) \vee 0)) \wedge 0 = x \wedge 0$ | [6390:9518] |
| 10144 | $(x \vee (y \wedge (x \vee (x \vee x)))) \wedge (c(y) \vee 1) = x$ | [8821 → 392] |

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|---|---|---|
| 10354 | $x \vee 1 = (y \wedge 1) \vee (c(y) \vee (x \vee 0))$ | [8877 → 15] |
| 10383 | $(x \wedge 1) \vee (c(x) \vee (y \vee 0)) = y \vee 1$ | [10354] |
| 10605 | $(x \wedge 1) \vee 1 = x \vee 1$ | [9085 → 1762,8565,9518,9616,8141,9380] |
| 10676,10675 | $c(x \vee x) \vee 1 = c(x) \vee 1$ | [662 → 10605,9228,7949] |
| 10714,10713 | $c(0 \vee (x \vee y)) \vee (x \vee 1) = c(y) \vee 1$ | [8055:10676] |
| 12015,12014 | $c(x) \vee 1 = 1$ | [8136 → 9199,10714] |
| 12035,12034 | $x \vee 1 = 1$ | [2 → 9199,12015] |
| 12056 | $(x \vee (y \wedge (x \vee (x \vee x)))) \wedge 1 = x$ | [10144:12035] |
| 12059,12058 | $(x \vee 0) \wedge 1 = x$ | [9397:12035,12035,12035,12,12035,12035] |
| 12067,12066 | $x \wedge 0 = 0$ | [8851:12035,45] |
| 12073,12072 | $1 \wedge (x \vee 0) = x$ | [7391:12035,12035,5913,12067,6383] |
| 12098 | $(x \vee (y \wedge x)) \wedge 1 = x$ | [390:12035] |
| 12162 | $(x \wedge 1) \vee (c(x) \vee (y \vee 0)) = 1$ | [10383:12035] |
| 12288 | $(x \vee c(c(x \vee 0) \vee (y \vee 0))) \wedge 1 = x$ | [8893:12035] |
| 12490 | $(x \vee c(y \vee (z \vee c(x)))) \wedge 1 = x$ | [1211:12035,12035] |
| 12518 | $(c(x) \vee 0) \vee (y \vee (x \wedge 1)) = 1$ | [137:12035] |
| 12524 | $c(x \vee x) = c(x \vee 0)$ | [8260:12059] |
| 12566 | $(x \vee c(y)) \wedge ((y \wedge 1) \vee (x \vee 0)) = x$ | [1877:12059] |
| 12607 | $c(x \vee 0) = c(0 \vee x)$ | [9495:12067,12073] |
| 12629 | $x \wedge 1 = 1 \wedge x$ | [7283:12067,12073] |
| 12680 | $c(x \vee x) \vee x = 1$ | [8207:12073] |
| 12761 | $1 \wedge x = x \wedge 1$ | [12629] |
| 12927,12926 | $c(x \vee (y \wedge z)) = (c(x \vee 0) \vee 0) \wedge (c(y) \vee c(z))$ | [12058 → 195] |
| 12932 | $c((x \wedge 1) \vee y) = (c(x) \vee 0) \wedge (c(y \vee 0) \vee 0)$ | [12058 → 195,12] |
| 12958 | $(c(x) \vee 0) \wedge (c(y \vee 0) \vee 0) = c((x \wedge 1) \vee y)$ | [12932] |
| 12967 | $x \vee c(x \vee 0) = 1$ | [12072 → 7743] |
| 13050 | $1 \wedge (x \vee c((0 \vee c((x \vee y) \vee (x \vee y))) \vee y)) = x$ | [440 → 12629] |
| 13191 | $(x \vee (x \vee x)) \wedge 1 = x$ | [8359 → 12761] |
| 13197 | $(0 \vee (x \vee (x \vee x))) \wedge 1 = x$ | [4217 → 12761] |
| 13253 | $(x \vee (y \wedge 1)) \wedge (x \vee (0 \vee c(y))) = x$ | [12761 → 66] |
| 13479 | $(x \vee (((x \vee c(y)) \vee (x \vee c(y))) \wedge y)) \wedge 1 = x$ | [12680 → 392] |
| 13483 | $(x \vee (((y \wedge 1) \vee (y \wedge 1)) \wedge (c(y) \vee 0))) \wedge 1 = x$ | [12680 → 388,12035] |
| 13491 | $(x \vee c(c((x \vee y) \vee (x \vee y)) \vee (z \vee y))) \wedge 1 = x$ | [12680 → 448,12035] |
| 13644,13643 | $x \wedge (c(x) \vee 0) = 0$ | [12967 → 669,5862,12059] |
| 13707 | $(x \vee (x \wedge 1)) \wedge 1 = x$ | [12761 → 12098] |
| 13751 | $(c(x \vee 0) \vee 0) \wedge (c(y) \vee c(x)) = c(x \vee 0)$ | [12098 → 84,12927,12,12059] |
| 13761 | $x \vee ((0 \vee c((x \vee y) \vee 0)) \vee y) = 1$ | [12524 → 250] |
| 13959 | $c(0 \vee c(x)) = x \wedge 1$ | [199 → 12607,6383,5476,12035] |
| 14074,14073 | $(0 \vee x) \wedge 1 = x$ | [12607 → 26,27,12059] |
| 14119,14118 | $x \vee (x \vee x) = x$ | [13197:14074] |
| 14142,14141 | $x \wedge 1 = x$ | [13191:14119] |
| 14146,14145 | $x \vee (y \wedge x) = x$ | [12056:14119,14142] |
| 14148,14147 | $1 \wedge x = x$ | [8359:14119] |
| 14152,14151 | $0 \vee x = x$ | [4217:14119,14148] |
| 14182,14181 | $c(c(x)) = x$ | [13959:14152,14142] |
| 14228,14227 | $x \vee x = x$ | [13707:14142,14142] |
| 14255 | $x \vee c(c(x \vee y) \vee (z \vee y)) = x$ | [13491:14228,14142] |
| 14258,14257 | $x \vee 0 = x$ | [13483:14142,14142,14228,13644,14142] |
| 14259 | $x \vee ((x \vee c(y)) \wedge y) = x$ | [13479:14228,14142] |
| 14279 | $(x \vee y) \wedge (x \vee c(y)) = x$ | [13253:14142,14152] |
| 14292,14291 | $c(x \vee y) = c(x) \wedge c(y)$ | [12958:14258,14258,14258,14142] |
| 14293 | $(x \vee c(y)) \wedge (y \vee x) = x$ | [12566:14142,14258] |
| 14295 | $c(x) \vee (y \vee x) = 1$ | [12518:14258,14142] |
| 14299 | $x \vee (c(y) \wedge (c(z) \wedge x)) = x$ | [12490:14292,14292,14182,14142] |

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|---|---|---|
| 14309 | $x \vee (x \wedge c(y)) = x$ | [12288:14258,14258,14292,14182,14142] |
| 14319 | $x \vee (c(x) \vee y) = 1$ | [12162:14142,14258] |
| 14349 | $(x \vee (c(y) \wedge z)) \wedge (y \vee (x \vee c(z))) = x$ | [3123:14258,14142] |
| 14351 | $(x \vee (c(y) \vee c(z))) \wedge ((y \wedge z) \vee x) = x$ | [665:14142,14258] |
| 14353 | $(x \vee y) \wedge (c(y) \vee x) = x$ | [424:14142,14258] |
| 14361 | $(x \vee c(y)) \vee (c(x) \wedge y) = 1$ | [234:14142,14258] |
| 14366,14365 | $c(x \wedge y) = c(x) \vee c(y)$ | [84:14258,14142] |
| 14369 | $x \vee ((c(x) \wedge c(y)) \vee y) = 1$ | [13761:14258,14292,14152] |
| 14373 | $c(x) \wedge (c(y) \vee c(x)) = c(x)$ | [13751:14258,14258,14258] |
| 14431 | $x \vee ((x \vee y) \wedge c(y)) = x$ | [13050:14228,14292,14152,14292,14366,14182,14182,14148] |
| 14464 | $x \vee ((x \vee y) \wedge (c(z) \wedge c(y))) = x$ | [14255:14292,14292,14366,14182,14182,14292] |
| 14527 | $x \vee (y \vee x) = y \vee x$ | [14227 → 2] |
| 14562 | $x \vee (y \vee z) = z \vee (y \vee x)$ | [14257 → 98,14258] |
| 14563 | $x \vee (y \vee z) = y \vee (z \vee x)$ | [14257 → 15,14258] |
| 14565 | $x \vee y = y \vee x$ | [14257 → 2,14258] |
| 14598 | $x \vee (y \vee (z \wedge x)) = y \vee x$ | [14145 → 2] |
| 14601,14600 | $(x \wedge y) \vee y = y$ | [14145 → 14565] |
| 14737 | $(c(x) \vee y) \wedge (y \vee x) = y$ | [14565 → 7] |
| 14741 | $(c(x) \vee c(y)) \wedge (y \wedge x) = 0$ | [14565 → 36] |
| 14829 | $x \vee ((y \wedge (x \vee z)) \vee z) = x \vee z$ | [2 → 14600] |
| 14857 | $c(x) \wedge ((y \wedge c(x)) \vee x) = y \wedge c(x)$ | [14600 → 7] |
| 14863 | $(x \vee y) \vee c(y) = 1$ | [14565 → 14295] |
| 14867 | $x \vee (x \wedge y) = x$ | [14181 → 14309] |
| 14880,14879 | $(x \wedge y) \vee x = x$ | [14565 → 14867] |
| 14907 | $x \vee (y \vee (x \wedge z)) = y \vee x$ | [14867 → 2] |
| 14965 | $(c(x) \vee y) \vee x = 1$ | [14565 → 14319] |
| 15141 | $(c(x) \vee y) \vee (z \vee x) = 1$ | [14965 → 2,12035] |
| 15281 | $x \vee ((c(y) \vee x) \wedge y) = x$ | [14565 → 14259] |
| 15300 | $x \vee ((c(x) \wedge y) \vee c(y)) = 1$ | [14259 → 14863,14366,14292,14182] |
| 15332 | $(x \vee y) \wedge (y \vee c(x)) = y$ | [14565 → 14279] |
| 15549 | $c(x) \wedge (x \vee (c(x) \wedge y)) = c(x) \wedge y$ | [14879 → 14293] |
| 15745 | $((x \vee y) \wedge c(y)) \vee x = x$ | [14565 → 14431] |
| 16074 | $(c(x) \vee y) \vee (x \vee z) = 1$ | [14565 → 15141] |
| 16201 | $x \vee (y \wedge (c(z) \wedge x)) = x$ | [14181 → 14299] |
| 16851 | $x \vee (y \wedge (z \wedge x)) = x$ | [14181 → 16201] |
| 16855 | $(x \wedge (y \wedge z)) \vee z = z$ | [14565 → 16851] |
| 16911,16910 | $(x \wedge y) \vee (y \vee z) = y \vee z$ | [14737 → 16855] |
| 17134 | $x \vee ((x \wedge c(y)) \wedge z) = x$ | [16074 → 14349,14292,14182,14142] |
| 17188 | $(x \wedge y) \vee (x \vee z) = x \vee z$ | [16074 → 14351,14182,14148,14182] |
| 17368 | $x \vee ((x \wedge y) \wedge z) = x$ | [14181 → 17134] |
| 17488 | $c(x) \vee (((x \wedge c(y)) \vee y) \wedge x) = 1$ | [15745 → 14361,14366,14292,14182,14182] |
| 17608 | $((x \wedge c(y)) \vee y) \vee x = (x \wedge c(y)) \vee y$ | [14369 → 15281,14182,14148,14182] |
| 17694 | $x \wedge (c(y) \vee x) = x$ | [14181 → 14373,14182,14182] |
| 17732 | $x \wedge (y \vee x) = x$ | [14181 → 17694] |
| 17784 | $(x \wedge y) \wedge y = x \wedge y$ | [14145 → 17732] |
| 18416,18415 | $(x \vee y) \vee z = y \vee (x \vee z)$ | [14565 → 14562] |
| 18569,18568 | $(x \wedge c(y)) \vee y = y \vee x$ | [17608:18416,14880] |
| 18926 | $c(x) \vee ((y \vee x) \wedge x) = 1$ | [17488:18569] |
| 18940 | $c(x) \wedge (x \vee y) = y \wedge c(x)$ | [14857:18569] |
| 18943 | $x \wedge c(y) = c(y) \wedge (y \vee x)$ | [18940] |
| 18987 | $x \vee (y \wedge c(x)) = x \vee y$ | [17784 → 18568,18569] |
| 19019 | $x \wedge (c(x) \vee y) = y \wedge x$ | [18568 → 15332,14182,14146,14182] |
| 19036,19035 | $(c(x) \vee y) \wedge x = y \wedge x$ | [18568 → 14293,14182,14146,14182] |
| 19045 | $x \wedge y = y \wedge (c(y) \vee x)$ | [19019] |

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| 19085 | $(x \vee y) \wedge y = y$ | [18926 → 14353,14601,14142] |
| 19111 | $(x \vee y) \wedge x = x$ | [18568 → 19085] |
| 19120,19119 | $x \wedge (y \wedge (z \wedge x)) = y \wedge (z \wedge x)$ | [16851 → 19085] |
| 19255 | $c(x) \vee (y \wedge x) = c(x) \vee y$ | [14181 → 18987] |
| 19297 | $x \wedge (y \vee c(x)) = y \wedge x$ | [14565 → 19019] |
| 19301,19300 | $(x \vee c(y)) \wedge y = y \wedge (x \vee c(y))$ | [14527 → 19019] |
| 19322 | $x \wedge y = y \wedge (x \vee c(y))$ | [19297] |
| 19355 | $x \vee (y \wedge (x \vee c(y))) = x$ | [14259:19301] |
| 19590 | $(c(x) \wedge (x \vee y)) \vee x = x \vee y$ | [19045 → 18568,14182] |
| 19634 | $(x \wedge (c(x) \vee y)) \vee y = y$ | [19045 → 14879] |
| 19852 | $(x \wedge y) \vee (y \wedge (c(y) \vee x)) = x \wedge y$ | [18568 → 19355,14182,14182] |
| 20391,20390 | $(x \vee (y \wedge z)) \wedge (x \vee z) = x \vee (y \wedge z)$ | [14598 → 17732] |
| 20402 | $(x \vee (y \wedge c(z))) \wedge z = z \wedge (x \vee c(z))$ | [14598 → 19035,19301] |
| 20556 | $(x \vee (c(y) \wedge z)) \wedge y = y \wedge (x \vee c(y))$ | [14907 → 19035,19301] |
| 20672 | $x \vee ((x \wedge y) \vee c(y)) = (x \wedge y) \vee c(y)$ | [15300 → 19634,14142,14182,14182] |
| 20674 | $((x \wedge y) \vee c(y)) \wedge x = x$ | [15300 → 19035,14148,14182] |
| 20678 | $((x \wedge (y \vee c(x))) \vee c(x)) \wedge y = y$ | [19322 → 20674] |
| 20707,20706 | $x \vee (c(x) \wedge y) = x \vee y$ | [14279 → 20674,14292,14182,20391] |
| 20713,20712 | $c(x) \wedge (x \vee y) = c(x) \wedge y$ | [15549:20707] |
| 20718 | $(c(x) \wedge y) \vee x = x \vee y$ | [19590:20713] |
| 20720 | $x \wedge c(y) = c(y) \wedge x$ | [18943:20713] |
| 20740 | $x \wedge y = y \wedge x$ | [14181 → 20720,14182] |
| 20742,20741 | $c(x) \wedge (y \vee x) = c(x) \wedge y$ | [19322 → 20720,14182] |
| 20768,20767 | $x \wedge (y \vee c(x)) = x \wedge y$ | [19322 → 20740] |
| 20770,20769 | $x \wedge (c(x) \vee y) = x \wedge y$ | [19045 → 20740] |
| 20775 | $((x \wedge y) \vee c(x)) \wedge y = y$ | [20678:20768] |
| 20778,20777 | $(x \vee (c(y) \wedge z)) \wedge y = y \wedge x$ | [20556:20768] |
| 20780,20779 | $(x \vee (y \wedge c(z))) \wedge z = z \wedge x$ | [20402:20768] |
| 20783 | $(x \wedge y) \vee (y \wedge x) = x \wedge y$ | [19852:20770] |
| 20960,20959 | $(x \vee c(y)) \wedge y = y \wedge x$ | [20740 → 20767] |
| 21072 | $(x \wedge y) \wedge (y \wedge x) = y \wedge x$ | [14741 → 20775,14292,14182,14182,14152] |
| 21087 | $x \vee ((y \wedge x) \vee c(y)) = (y \wedge x) \vee c(y)$ | [20775 → 14879] |
| 21465,21464 | $x \vee ((y \wedge x) \vee z) = x \vee z$ | [2 → 16910] |
| 21469,21468 | $(x \wedge y) \vee c(x) = y \vee c(x)$ | [21087:21465] |
| 21588,21587 | $x \vee ((x \wedge y) \vee z) = x \vee z$ | [2 → 17188] |
| 21594,21593 | $(x \wedge y) \vee c(y) = x \vee c(y)$ | [20672:21588] |
| 22207 | $(x \wedge c(y)) \vee (x \wedge y) = x$ | [20769 → 19255,14292,14182,14292,14182,14880] |
| 22209 | $(c(x) \wedge y) \vee (y \wedge x) = y$ | [20767 → 19255,14292,14182,14292,14182,14601] |
| 22240 | $(x \wedge c(y)) \vee (y \wedge x) = x$ | [20740 → 22207] |
| 22267 | $(x \wedge y) \vee (y \wedge c(x)) = y$ | [14181 → 22209] |
| 22271 | $(x \wedge y) \vee (c(y) \wedge x) = x$ | [14565 → 22209] |
| 22394 | $(x \wedge y) \vee (c(x) \wedge y) = y$ | [20740 → 22267] |
| 22462 | $x \vee ((x \vee y) \wedge (z \wedge c(y))) = x$ | [14181 → 14464] |
| 23045,23044 | $(x \wedge y) \vee ((y \wedge x) \vee z) = (y \wedge x) \vee z$ | [20783 → 18415] |
| 23048 | $x \vee (y \wedge z) = (z \wedge y) \vee x$ | [20783 → 14563,23045] |
| 23273 | $(x \wedge y) \vee ((y \wedge x) \wedge z) = x \wedge y$ | [21072 → 17368] |
| 23304,23303 | $(x \vee (y \vee c(z))) \wedge (z \vee x) = (y \wedge z) \vee x$ | [21464 → 14293,18416,21594] |
| 23439,23438 | $((c(x) \wedge y) \vee z) \wedge x = z \wedge x$ | [21587 → 19035,19036] |
| 24841 | $x \wedge (y \wedge (x \vee c(z))) = y \wedge x$ | [18568 → 20777,23439,14366,14182] |
| 25350 | $x \vee ((y \vee x) \wedge (z \wedge c(y))) = x$ | [14565 → 22462] |
| 26809,26808 | $x \wedge (y \wedge (x \vee z)) = y \wedge x$ | [14181 → 24841] |
| 26840 | $(x \wedge y) \wedge (z \wedge x) = z \wedge (x \wedge y)$ | [22271 → 26808] |
| 26851 | $x \wedge (y \wedge (z \vee x)) = y \wedge x$ | [14907 → 26808] |
| 26869 | $x \wedge ((x \vee y) \wedge z) = x \wedge z$ | [20959 → 26808,14292,20778] |

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| 26872 | $(x \wedge (y \vee z)) \wedge y = x \wedge y$ | [17784 → 26808,26809] |
| 26983,26982 | $(x \wedge (y \vee z)) \vee c(y) = x \vee c(y)$ | [26808 → 21468,21594] |
| 27026 | $x \wedge ((y \vee x) \wedge z) = x \wedge z$ | [20959 → 26851,14292,20780] |
| 27096 | $x \vee (y \wedge (z \vee c(x))) = x \vee y$ | [26851 → 20718,18569] |
| 27140 | $(x \wedge (y \vee z)) \vee c(z) = x \vee c(z)$ | [26851 → 21468,21594] |
| 27178,27177 | $(x \wedge y) \wedge (y \wedge z) = (x \wedge y) \wedge z$ | [22394 → 26869] |
| 27266 | $(((x \vee y) \wedge z) \wedge c(x)) \vee (x \wedge z) = (x \vee y) \wedge z$ | [26869 → 22240] |
| 27342,27341 | $(x \wedge y) \wedge z = x \wedge (y \wedge z)$ | [22271 → 26872,27178] |
| 27373 | $((x \vee y) \wedge (z \wedge c(x))) \vee (x \wedge z) = (x \vee y) \wedge z$ | [27266:27342] |
| 27452 | $x \wedge (y \wedge z) = y \wedge (z \wedge x)$ | [26840:27342,19120] |
| 27776 | $(x \wedge y) \vee (y \wedge (x \wedge z)) = x \wedge y$ | [23273:27342] |
| 28022,28021 | $((x \vee y) \wedge z) \vee c(y) = z \vee c(y)$ | [27026 → 21468,21469] |
| 28503,28502 | $(x \vee y) \wedge (z \wedge c(x)) = y \wedge (z \wedge c(x))$ | [25350 → 14293,28022,20960] |
| 28505 | $(x \wedge (y \wedge c(z))) \vee (z \wedge y) = (z \vee x) \wedge y$ | [27373:28503] |
| 28507 | $(x \vee y) \wedge z = (y \wedge (z \wedge c(x))) \vee (x \wedge z)$ | [28505] |
| 28888 | $((x \vee c(y)) \wedge z) \vee y = y \vee z$ | [23048 → 27096] |
| 28890 | $(x \wedge (y \vee c(z))) \vee z = z \vee x$ | [14565 → 27096] |
| 30272,30271 | $(x \wedge (y \wedge (z \vee c(u)))) \vee u = u \vee (x \wedge y)$ | [27452 → 28888] |
| 30647 | $(x \wedge (y \vee z)) \vee z = (x \wedge y) \vee z$ | [14829 → 14293,18416,26983,23304] |
| 32813 | $(x \vee c(y)) \wedge (y \vee (x \wedge (z \vee y))) = x \wedge (z \vee y)$ | [27140 → 14353,14182] |
| 33040 | $(x \wedge ((x \wedge y) \vee z)) \vee (x \wedge y) = x \wedge ((x \wedge y) \vee z)$ | [19111 → 27776] |
| 33573,33572 | $(x \wedge (y \vee z)) \vee y = y \vee (x \wedge z)$ | [28890 → 30647,30272] |
| 33587,33586 | $(x \wedge (y \wedge c(z))) \vee (z \wedge y) = (x \wedge y) \vee (z \wedge y)$ | [22240 → 30647] |
| 33624,33623 | $x \vee (y \wedge (z \vee x)) = (y \wedge z) \vee x$ | [14565 → 30647] |
| 33638,33637 | $x \wedge ((x \wedge y) \vee z) = (x \wedge y) \vee (x \wedge z)$ | [33040:33573] |
| 33646,33645 | $(x \vee y) \wedge z = (y \wedge z) \vee (x \wedge z)$ | [28507:33587] |
| 33672 | $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ | [32813:33624,33646,20742,33638,16911] |
| 33674 | $\$Ans(DIST1)$ | [33672,13] |

**Proof BA2**

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|---|---|---|
| 2 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [] |
| 3 | $x \wedge y = c(c(x) \vee c(y))$ | [] |
| 4 | $c(c(x) \vee c(y)) = x \wedge y$ | [3] |
| 7 | $(x \vee c(y)) \wedge (x \vee y) = x$ | [] |
| 10,9 | $x \vee c(x) = 1$ | [] |
| 12,11 | $c(1) = 0$ | [] |
| 13 | $A \vee (B \wedge (A \vee C)) \neq A \vee (C \wedge (A \vee B)) \mid \$Ans(MOD)$ | [] |
| 14 | $A \vee (C \wedge (A \vee B)) \neq A \vee (B \wedge (A \vee C)) \mid \$Ans(MOD)$ | [13] |
| 15 | $x \vee (y \vee (z \vee u)) = z \vee (x \vee (y \vee u))$ | [2 → 2] |
| 16 | $x \vee (y \vee (z \vee u)) = y \vee (z \vee (x \vee u))$ | [15] |
| 18,17 | $1 \vee 0 = 1$ | [11 → 9] |
| 19 | $x \vee (y \vee c(x)) = y \vee 1$ | [9 → 2] |
| 21 | $x \vee 1 = 1 \vee (x \vee 0)$ | [17 → 2] |
| 22 | $1 \vee (x \vee 0) = x \vee 1$ | [21] |
| 23 | $c(0 \vee c(x)) = 1 \wedge x$ | [11 → 4] |
| 25 | $c((x \wedge y) \vee c(z)) = (c(x) \vee c(y)) \wedge z$ | [4 → 4] |
| 28,27 | $c(c(x) \vee 0) = x \wedge 1$ | [11 → 4] |
| 30,29 | $c(c(x) \vee (y \wedge z)) = x \wedge (c(y) \vee c(z))$ | [4 → 4] |
| 31 | $x \wedge c(x) = 0$ | [9 → 4,12] |
| 33 | $(c(x) \vee c(y)) \vee (x \wedge y) = 1$ | [4 → 9] |
| 35 | $1 \wedge 0 = 0$ | [11 → 31] |

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| 37 | $(c(x) \vee c(y)) \wedge (x \wedge y) = 0$ | [4 → 31] |
| 39 | $x \vee (1 \vee (y \vee 0)) = y \vee (x \vee 1)$ | [21 → 2] |
| 40 | $x \vee (y \vee 1) = y \vee (1 \vee (x \vee 0))$ | [39] |
| 42,41 | $(x \vee 0) \wedge (x \vee 1) = x$ | [11 → 7] |
| 43 | $(x \vee (y \wedge z)) \wedge (x \vee (c(y) \vee c(z))) = x$ | [4 → 7] |
| 46,45 | $1 \wedge (x \vee x) = x$ | [9 → 7] |
| 47 | $(x \vee 0) \wedge (1 \vee (x \vee 0)) = x$ | [21 → 7,12] |
| 49 | $(1 \vee c(0)) \wedge 1 = 1$ | [17 → 7] |
| 51 | $(x \vee c(c(x))) \wedge 1 = x$ | [9 → 7] |
| 53 | $(x \vee c(y \vee z)) \wedge (y \vee (x \vee z)) = x$ | [2 → 7] |
| 56,55 | $1 \wedge (x \vee ((x \vee y) \vee y)) = x \vee y$ | [2 → 45] |
| 59 | $x \vee (y \vee 1) = 1 \vee (x \vee (y \vee 0))$ | [22 → 2] |
| 60 | $1 \vee (x \vee (y \vee 0)) = x \vee (y \vee 1)$ | [59] |
| 64,63 | $c(0 \vee 0) = 1 \wedge 1$ | [11 → 23] |
| 65 | $c(0 \vee (x \wedge y)) = 1 \wedge (c(x) \vee c(y))$ | [4 → 23] |
| 67 | $(x \vee (1 \wedge y)) \wedge (x \vee (0 \vee c(y))) = x$ | [23 → 7] |
| 71 | $(0 \vee c(x)) \vee (1 \wedge x) = 1$ | [23 → 9] |
| 73 | $(x \vee (1 \wedge 1)) \wedge (x \vee (0 \vee 0)) = x$ | [63 → 7] |
| 86,85 | $c((x \wedge y) \vee 0) = (c(x) \vee c(y)) \wedge 1$ | [4 → 27] |
| 91 | $(c(x) \vee 0) \vee (x \wedge 1) = 1$ | [27 → 9] |
| 95 | $x \vee (y \vee (z \vee c(x))) = y \vee (z \vee 1)$ | [9 → 15] |
| 99 | $x \vee (y \vee (z \vee u)) = z \vee (y \vee (x \vee u))$ | [2 → 15] |
| 106 | $(x \vee c(y \vee (z \vee u))) \wedge (z \vee (x \vee (y \vee u))) = x$ | [15 → 7] |
| 128 | $1 \wedge (x \vee (y \vee ((x \vee (y \vee z)) \vee z))) = x \vee (y \vee z)$ | [16 → 45] |
| 131 | $(x \vee c(y \vee (z \vee u))) \wedge (y \vee (z \vee (x \vee u))) = x$ | [16 → 7] |
| 138 | $(c(x) \vee 0) \vee (y \vee (x \wedge 1)) = y \vee 1$ | [27 → 19] |
| 142 | $(c(x) \vee c(y)) \vee (z \vee (x \wedge y)) = z \vee 1$ | [4 → 19] |
| 162,161 | $c(x \vee c(y)) = (c(x \vee 0) \vee c(x \vee 1)) \wedge y$ | [41 → 25] |
| 164,163 | $((1 \wedge 1) \vee c(0 \vee 1)) \wedge x = 1 \wedge x$ | [35 → 25,162,64,12,10] |
| 175 | $(0 \vee (1 \wedge 1)) \vee 0 = 1$ | [63 → 71,46] |
| 179 | $(0 \vee c(x \vee x)) \vee x = 1$ | [45 → 71] |
| 183 | $1 \wedge ((0 \vee (1 \wedge 1)) \vee 1) = 0 \vee (1 \wedge 1)$ | [175 → 41] |
| 186,185 | $(0 \vee (1 \wedge 1)) \vee 1 = 1 \vee 1$ | [175 → 22] |
| 192 | $0 \vee (1 \wedge 1) = 1$ | [183:186,46] |
| 197,196 | $c((x \wedge 1) \vee (y \wedge z)) = (c(x) \vee 0) \wedge (c(y) \vee c(z))$ | [27 → 29] |
| 198 | $c(c(x) \vee 1) = x \wedge (((0 \vee c(1 \vee 1)) \wedge 0) \vee 0)$ | [49 → 29,162,18,12,12] |
| 200 | $c(c(x) \vee y) = x \wedge (0 \vee c(y \vee y))$ | [45 → 29,12] |
| 201 | $c(c(x) \vee y) = x \wedge (c(y \vee 0) \vee c(y \vee 1))$ | [41 → 29] |
| 202 | $x \wedge (0 \vee c(y \vee y)) = c(c(x) \vee y)$ | [200] |
| 203 | $x \wedge (c(y \vee 0) \vee c(y \vee 1)) = c(c(x) \vee y)$ | [201] |
| 206 | $(c(x) \vee (y \wedge z)) \vee (x \wedge (c(y) \vee c(z))) = 1$ | [29 → 9] |
| 211 | $(0 \vee c(1 \wedge 1)) \wedge 1 = 0$ | [192 → 7] |
| 213 | $0 \vee (x \vee (1 \wedge 1)) = x \vee 1$ | [192 → 2] |
| 227 | $(((c(x \vee 0) \vee c(x \vee 1)) \wedge c(x)) \vee 0) \vee x = 1$ | [51 → 91,162] |
| 235 | $((x \wedge 1) \vee c(y)) \vee ((c(x) \vee 0) \wedge y) = 1$ | [27 → 33] |
| 241 | $(c(x \vee 0) \vee c(x \vee 1)) \vee x = 1$ | [41 → 33] |
| 251 | $x \vee ((0 \vee c((x \vee y) \vee (x \vee y))) \vee y) = 1$ | [2 → 179] |
| 256 | $x \vee (y \vee 1) = (0 \vee c(z \vee z)) \vee (x \vee (y \vee z))$ | [179 → 15] |
| 259 | $x \vee 1 = (0 \vee c(y \vee y)) \vee (x \vee y)$ | [179 → 2] |
| 261 | $(0 \vee c(x \vee x)) \vee (y \vee (z \vee x)) = y \vee (z \vee 1)$ | [256] |
| 264,263 | $((1 \wedge (1 \wedge 1)) \vee 0) \vee 0 = 1$ | [211 → 91,162,64,164] |
| 279 | $c(c(x) \vee y) = x \wedge (c(y \vee 0) \vee c(1 \vee (y \vee 0)))$ | [47 → 29] |
| 288,287 | $(0 \vee 0) \vee 1 = 0 \vee 1$ | [91 → 213,12] |
| 307 | $(x \vee c(1 \vee (y \vee 0))) \wedge (y \vee (x \vee 1)) = x$ | [39 → 7] |

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| 313,312 | $(1 \wedge (1 \wedge 1)) \vee 0 = 1$ | [263 → 47,264,46] |
| 315 | $1 \wedge (1 \wedge 1) = 1$ | [312 → 47,313,46] |
| 323 | $x \vee (y \vee 1) = x \vee (1 \vee (y \vee 0))$ | [2 → 40] |
| 326 | $x \vee (1 \vee (y \vee 0)) = x \vee (y \vee 1)$ | [323] |
| 335 | $(x \vee c(y \vee 1)) \wedge (y \vee (1 \vee (x \vee 0))) = x$ | [40 → 7] |
| 377 | $(x \vee y) \wedge (x \vee (0 \vee c(y \vee y))) = x$ | [45 → 43,12] |
| 381 | $1 \wedge (0 \vee (0 \vee 0)) = 0$ | [192 → 43,12,12] |
| 389 | $(x \vee (y \wedge (c(z) \vee 0))) \wedge (x \vee (c(y) \vee (z \wedge 1))) = x$ | [27 → 43] |
| 391 | $(x \vee (y \wedge x)) \wedge (c(y) \vee 1) = x$ | [19 → 43] |
| 393 | $(x \vee (y \wedge z)) \wedge (c(y) \vee (x \vee c(z))) = x$ | [2 → 43] |
| 400,399 | $(0 \vee c(0 \vee (0 \vee 0))) \vee 0 = 1$ | [381 → 71] |
| 423 | $(x \vee (1 \wedge 1)) \wedge (0 \vee (x \vee 0)) = x$ | [63 → 53] |
| 425 | $(x \vee (y \wedge 1)) \wedge (c(y) \vee (x \vee 0)) = x$ | [27 → 53] |
| 441 | $(x \vee c((0 \vee c((x \vee y) \vee (x \vee y))) \vee y)) \wedge 1 = x$ | [179 → 53] |
| 445 | $(1 \vee c(x \vee (y \vee 0))) \wedge (y \vee (x \vee 1)) = 1$ | [39 → 53] |
| 449 | $(x \vee c(y \vee (z \vee u))) \wedge (z \vee (y \vee (x \vee u))) = x$ | [15 → 53] |
| 467 | $((x \vee x) \vee x) \wedge (0 \vee 1) = x \vee x$ | [45 → 391,12] |
| 501 | $1 \wedge (x \vee (y \vee ((y \vee (x \vee z)) \vee z))) = y \vee (x \vee z)$ | [15 → 55] |
| 506 | $0 \vee c(0 \vee (0 \vee 0)) = 1$ | [399 → 47,400,46] |
| 521 | $0 \vee (x \vee c(0 \vee (0 \vee 0))) = x \vee 1$ | [506 → 2] |
| 562,561 | $(0 \vee c((x \vee 0) \vee (x \vee 0))) \vee (x \vee 1) = 1 \vee 1$ | [179 → 60] |
| 581 | $1 \wedge (c(x \vee 0) \vee c(1 \vee (x \vee 0))) = c(0 \vee x)$ | [47 → 65] |
| 583 | $c(0 \vee x) = 1 \wedge (0 \vee c(x \vee x))$ | [45 → 65,12] |
| 585 | $1 \wedge (0 \vee c(x \vee x)) = c(0 \vee x)$ | [583] |
| 619 | $(x \vee (1 \wedge y)) \wedge (0 \vee (x \vee c(y))) = x$ | [2 → 67] |
| 632,631 | $((1 \wedge 1) \vee c(0 \vee 1)) \vee 0 = 1$ | [63 → 241] |
| 644 | $(1 \wedge 1) \vee c(0 \vee 1) = 1$ | [241 → 47,64,632,46,64] |
| 659 | $c(x \vee 0) = (0 \vee c(x \vee x)) \wedge 1$ | [45 → 85,12] |
| 663 | $(0 \vee c(x \vee x)) \wedge 1 = c(x \vee 0)$ | [659] |
| 666 | $(x \vee ((c(y) \vee c(z)) \wedge 1)) \wedge ((y \wedge z) \vee (x \vee 0)) = x$ | [85 → 53] |
| 670 | $((x \wedge y) \vee 0) \wedge ((c(x) \vee c(y)) \wedge 1) = 0$ | [85 → 31] |
| 684 | $(1 \wedge 1) \vee c(1 \vee (0 \vee 0)) = 1$ | [21 → 644] |
| 686 | $1 \wedge (0 \vee ((1 \wedge 1) \vee 1)) = 1 \wedge 1$ | [644 → 53] |
| 689,688 | $(1 \wedge 1) \vee 1 = (0 \vee 1) \vee 1$ | [644 → 19] |
| 704,703 | $1 \wedge 1 = 0 \vee 1$ | [686:689,56] |
| 711 | $(0 \vee 1) \vee c(1 \vee (0 \vee 0)) = 1$ | [684:704] |
| 733 | $(x \vee (0 \vee 1)) \wedge (0 \vee (x \vee 0)) = x$ | [423:704] |
| 756,755 | $1 \wedge (0 \vee 1) = 1$ | [315:704] |
| 773 | $0 \vee (x \vee (0 \vee 1)) = x \vee 1$ | [213:704] |
| 779 | $0 \vee (0 \vee 1) = 1$ | [192:704] |
| 789 | $(x \vee (0 \vee 1)) \wedge (x \vee (0 \vee 0)) = x$ | [73:704] |
| 792,791 | $c(0 \vee 0) = 0 \vee 1$ | [63:704] |
| 793 | $(1 \vee (0 \vee 1)) \wedge (0 \vee 1) = 1$ | [703 → 391,12] |
| 795 | $c(c(x) \vee (0 \vee 1)) = x \wedge (0 \vee 0)$ | [703 → 29,12,12] |
| 807 | $0 \vee (1 \vee (0 \vee 0)) = 1$ | [21 → 779] |
| 812,811 | $0 \vee (0 \vee (x \vee 1)) = x \vee 1$ | [779 → 16] |
| 815 | $c((0 \vee 1) \vee (x \wedge y)) = (0 \vee 0) \wedge (c(x) \vee c(y))$ | [791 → 29] |
| 841 | $0 \vee (1 \vee (x \vee (0 \vee 0))) = x \vee 1$ | [807 → 16] |
| 861 | $0 \vee (0 \vee (1 \vee (x \vee 0))) = x \vee 1$ | [40 → 773] |
| 863 | $x \vee 1 = 0 \vee (0 \vee (1 \vee (x \vee 0)))$ | [861] |
| 867,866 | $(0 \vee (1 \vee 1)) \wedge (0 \vee 1) = 1$ | [40 → 793,18] |
| 926,925 | $(1 \vee (0 \vee 0)) \vee 1 = (0 \vee 1) \vee 1$ | [711 → 19] |
| 1074,1073 | $1 \wedge (0 \vee c((x \vee y) \vee (x \vee y))) = c(x \vee (0 \vee y))$ | [2 → 583] |
| 1188 | $(0 \vee (x \vee 1)) \wedge (0 \vee (x \vee 0)) = x$ | [2 → 733] |

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| 1212 | $(x \vee c(y \vee (z \vee c(x)))) \wedge (z \vee (y \vee 1)) = x$ | [19 → 106] |
| 1230 | $(0 \vee (x \vee 1)) \wedge (x \vee (0 \vee 0)) = x$ | [2 → 789] |
| 1233,1232 | $0 \vee c((0 \vee 0) \vee (0 \vee 0)) = (1 \vee 1) \wedge 1$ | [179 → 789,562] |
| 1295,1294 | $(c((0 \vee 0) \vee 0) \vee c(0 \vee 1)) \vee 1 = 0 \vee (1 \vee 1)$ | [241 → 841,288] |
| 1297,1296 | $((1 \vee 1) \wedge 1) \vee 1 = 0 \vee (1 \vee 1)$ | [179 → 841,1233] |
| 1319 | $c(x \vee 1) = c(0 \vee (0 \vee (1 \vee (x \vee 0))))$ | [861 → 583,1074] |
| 1326,1325 | $0 \vee (0 \vee (1 \vee (x \vee 0))) = 1 \vee (x \vee 0)$ | [21 → 863] |
| 1328 | $c(x \vee 1) = c(1 \vee (x \vee 0))$ | [1319:1326] |
| 1441 | $c((0 \vee 0) \vee 0) \vee c(0 \vee 1) = (1 \vee 1) \wedge 1$ | [241 → 1230,288,1295,812,288] |
| 1590,1589 | $c(1 \vee (c(x) \vee 0)) = x \wedge (0 \vee c(1 \vee 1))$ | [21 → 200] |
| 1625 | $c((1 \wedge c(x \vee x)) \vee x) = 0$ | [7 → 202,162,792,10] |
| 1684 | $((1 \wedge c(x \vee x)) \vee x) \vee 0 = 1$ | [1625 → 9] |
| 1705,1704 | $x \vee (((1 \wedge c(y \vee y)) \vee y) \vee 1) = x \vee (1 \vee 1)$ | [1684 → 326] |
| 1706 | $(1 \wedge c(x \vee x)) \vee x = 1$ | [1684 → 1188,1705,867] |
| 1756 | $x \vee 1 = (1 \wedge c(y \vee y)) \vee (x \vee y)$ | [1706 → 2] |
| 1763 | $(1 \wedge c(x \vee x)) \vee (y \vee x) = y \vee 1$ | [1756] |
| 1868 | $(x \vee y) \wedge (0 \vee (x \vee c(y \vee y))) = x$ | [2 → 377] |
| 1878 | $(x \vee ((c(y) \vee 0) \wedge 1)) \wedge ((y \wedge 1) \vee (x \vee 0)) = x$ | [27 → 425] |
| 1912 | $(0 \vee (0 \vee 0)) \vee (x \vee 1) = 0 \vee (x \vee 1)$ | [521 → 95] |
| 2368 | $(c(x) \vee (1 \wedge y)) \vee (x \wedge (0 \vee c(y))) = 1$ | [11 → 206] |
| 2370 | $(0 \vee (x \wedge x)) \vee c(x) = 1$ | [45 → 206,12] |
| 2379,2378 | $(0 \vee (x \wedge x)) \vee 1 = x \vee 1$ | [2370 → 19] |
| 2397 | $(0 \vee (x \wedge x)) \vee (y \vee c(x)) = y \vee 1$ | [2370 → 2] |
| 2404,2403 | $1 \vee ((0 \vee (x \wedge x)) \vee 0) = x \vee 1$ | [21 → 2378] |
| 2463,2462 | $x \vee (((1 \wedge c(0)) \vee 0) \vee 1) = x \vee (1 \vee 1)$ | [227 → 326,792,10] |
| 2464 | $(1 \wedge c(0)) \vee 0 = 1$ | [227 → 1188,792,10,2463,867,792,10] |
| 2469,2468 | $(1 \wedge c(0)) \vee 1 = 1 \vee 1$ | [2464 → 1763,792,756] |
| 2480 | $1 \wedge c(0) = 1$ | [2464 → 1188,2469,867] |
| 2485,2484 | $(c(0) \vee 1) \wedge (0 \vee 1) = c(0)$ | [2480 → 391,12] |
| 2486 | $(x \vee 1) \wedge (0 \vee (x \vee c(c(0)))) = x$ | [2480 → 619] |
| 2533 | $(1 \vee (c(0) \vee 0)) \wedge (0 \vee 1) = c(0)$ | [21 → 2484] |
| 2565 | $0 \vee (c(0) \wedge c(0)) = c(0)$ | [2370 → 2486,2379,2485] |
| 2584 | $(0 \vee c(c(0) \wedge c(0))) \wedge c(0) = 0$ | [2565 → 7] |
| 2606 | $((0 \wedge (0 \vee c(1 \vee 1))) \vee c(0 \vee 1)) \wedge 1 = 0 \wedge 1$ | [2533 → 85,28,1590] |
| 2614 | $((0 \wedge (0 \vee c(1 \vee 1))) \vee c(0 \vee 1)) \vee c(0) = 1$ | [2533 → 33,1590] |
| 3124 | $(x \vee ((c(y) \vee 0) \wedge z)) \wedge ((y \wedge 1) \vee (x \vee c(z))) = x$ | [27 → 393] |
| 3345 | $0 \vee ((x \vee 1) \wedge (x \vee 1)) = x \vee 1$ | [2370 → 335,2404,56] |
| 3354,3353 | $(c(x \vee 1) \vee c(x \vee 1)) \vee (x \vee 1) = 0 \vee 1$ | [3345 → 142] |
| 3542 | $(0 \vee (0 \vee 0)) \vee (0 \vee (1 \vee 1)) = 1 \vee 1$ | [1296 → 1912,1297,812] |
| 3896 | $((1 \wedge c(x)) \vee (1 \wedge x)) \vee 0 = 1$ | [7 → 2368,162,792,10] |
| 3925,3924 | $((1 \wedge c(x)) \vee (1 \wedge x)) \vee 1 = 1 \vee 1$ | [3896 → 1763,792,756] |
| 3929 | $(1 \wedge c(x)) \vee (1 \wedge x) = 1$ | [3896 → 1188,3925,867] |
| 3965 | $(1 \wedge x) \vee c(0 \vee x) = 1$ | [585 → 3929,162,792,10,46] |
| 4034 | $x \vee c(0 \vee (x \vee x)) = 1$ | [45 → 3965] |
| 4113,4112 | $(1 \wedge x) \vee 1 = (0 \vee x) \vee 1$ | [3965 → 2397,2379] |
| 4205 | $x \vee c(x \vee (0 \vee x)) = 1$ | [2 → 4034] |
| 4219,4218 | $1 \wedge (0 \vee (x \vee (x \vee x))) = x$ | [4034 → 131] |
| 4220 | $1 \wedge (x \vee (x \vee (0 \vee x))) = x$ | [4034 → 106] |
| 4229 | $(0 \vee (x \vee x)) \vee 1 = x \vee 1$ | [4034 → 2397,2379] |
| 4301,4300 | $(x \vee (0 \vee x)) \vee 1 = x \vee 1$ | [4205 → 2397,2379] |
| 4438,4437 | $1 \wedge (c(0) \vee (c(0) \vee 1)) = c(0)$ | [95 → 4218] |
| 4475 | $1 \wedge ((1 \vee (0 \vee 0)) \vee ((0 \vee 1) \vee 1)) = 1 \vee (0 \vee 0)$ | [807 → 4220,926] |
| 4555 | $(c(0) \vee 1) \vee 1 = c(0) \vee 1$ | [19 → 4229] |
| 4568,4567 | $0 \vee (1 \vee 1) = 1$ | [4229 → 128,4219] |

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| 4591,4590 | $1 \vee 1 = 0 \vee 1$ | [3542:4568,4301] |
| 4879 | $((0 \wedge (0 \vee c(0 \vee 1))) \vee c(0 \vee 1)) \vee c(0) = 1$ | [2614:4591] |
| 4885 | $((0 \wedge (0 \vee c(0 \vee 1))) \vee c(0 \vee 1)) \wedge 1 = 0 \wedge 1$ | [2606:4591] |
| 4946,4945 | $c(1 \vee (c(x) \vee 0)) = x \wedge (0 \vee c(0 \vee 1))$ | [1589:4591] |
| 4949 | $c((0 \vee 0) \vee 0) \vee c(0 \vee 1) = (0 \vee 1) \wedge 1$ | [1441:4591] |
| 4977 | $c(c(x) \vee 1) = x \wedge (((0 \vee c(0 \vee 1)) \wedge 0) \vee 0)$ | [198:4591] |
| 4983 | $(0 \vee c(x \vee x)) \vee (1 \vee x) = 0 \vee 1$ | [259 → 4590] |
| 5311,5310 | $c(0) \vee 1 = c(0)$ | [4555 → 55,4438] |
| 5348 | $c(c(0)) = 0 \wedge (0 \vee c(0 \vee 1))$ | [4555 → 1328,5311,5311,4946] |
| 5359,5358 | $c(0) = 0 \vee 1$ | [4555 → 811,5311,10,5311,5311] |
| 5407,5406 | $0 \wedge (0 \vee c(0 \vee 1)) = c(0 \vee 1)$ | [5348:5359] |
| 5425,5424 | $0 \vee 1 = 1$ | [4879:5407,5359,3354] |
| 5441,5440 | $(0 \vee 0) \wedge 1 = 0$ | [2584:5359,5425,5359,5425,704,5425,12,5359,5425] |
| 5449 | $((0 \wedge (0 \vee 0)) \vee 0) \wedge 1 = 0 \wedge 1$ | [4885:5425,12,5425,12] |
| 5457,5456 | $1 \vee (0 \vee 0) = 1$ | [4475:5425,4591,5425,926,5425,4591,5425,704,5425] |
| 5467,5466 | $0 \wedge (0 \vee 0) = 0$ | [5406:5425,12,5425,12] |
| 5477,5476 | $c(0) = 1$ | [5358:5425] |
| 5500 | $(0 \vee c(x \vee x)) \vee (1 \vee x) = 1$ | [4983:5425] |
| 5505,5504 | $c(c(x) \vee 1) = x \wedge (((0 \vee 0) \wedge 0) \vee 0)$ | [4977:5425,12] |
| 5532 | $c((0 \vee 0) \vee 0) \vee 0 = 1$ | [4949:5425,12,5425,704,5425] |
| 5536 | $c(1 \vee (c(x) \vee 0)) = x \wedge (0 \vee 0)$ | [4945:5425,12] |
| 5653,5652 | $1 \vee 1 = 1$ | [4590:5425] |
| 5839,5838 | $c(1 \vee (x \wedge y)) = (0 \vee 0) \wedge (c(x) \vee c(y))$ | [815:5425] |
| 5841,5840 | $x \wedge (((0 \vee 0) \wedge 0) \vee 0) = x \wedge (0 \vee 0)$ | [795:5425,5505] |
| 5843,5842 | $c(0 \vee 0) = 1$ | [791:5425] |
| 5863,5862 | $1 \wedge 1 = 1$ | [703:5425] |
| 5875,5874 | $((x \vee x) \vee x) \wedge 1 = x \vee x$ | [467:5425] |
| 5914,5913 | $0 \wedge 1 = 0$ | [5449:5467,5441] |
| 5949 | $c(c(x) \vee 1) = x \wedge (0 \vee 0)$ | [5504:5841] |
| 5967 | $(x \vee (0 \wedge x)) \wedge 1 = x$ | [5476 → 391,5653] |
| 6032,6031 | $(0 \vee 0) \wedge (c(x) \vee c(y)) = 0 \wedge (c(x) \vee c(y))$ | [5476 → 29,5839] |
| 6054,6053 | $c(1 \vee (x \wedge y)) = 0 \wedge (c(x) \vee c(y))$ | [5838:6032] |
| 6258,6257 | $1 \wedge (c(x \vee (0 \wedge x)) \vee 0) = c(0 \vee x)$ | [5967 → 65,12] |
| 6384,6383 | $0 \vee 0 = 0$ | [5532 → 279,12,5843,5457,12,18,5875] |
| 6392,6391 | $c(c(x) \vee 1) = x \wedge 0$ | [5949:6384] |
| 6438,6437 | $c(1 \vee (c(x) \vee 0)) = x \wedge 0$ | [5536:6384] |
| 6500,6499 | $0 \vee (x \vee 0) = x \vee 0$ | [6383 → 2] |
| 6639 | $(1 \wedge (x \vee 0)) \vee c(x \vee 0) = 1$ | [6499 → 3965] |
| 6783 | $c(1 \vee ((x \wedge 0) \vee 0)) = (1 \vee (c(x) \vee 0)) \wedge 0$ | [6437 → 6437] |
| 6785 | $(1 \vee (x \wedge 0)) \wedge (c(x) \vee 1) = 1$ | [6437 → 445,5653] |
| 7015 | $(0 \vee (((x \wedge 1) \vee (x \wedge 0)) \wedge x)) \vee 1 = x \vee 1$ | [5500 → 95,162,28,6392,5653] |
| 7284 | $1 \wedge ((x \wedge 1) \vee (x \wedge 0)) = 1 \wedge x$ | [279 → 581,6384,5477,6384,18,12,18,6438,162,6384,5477,5425,12,18] |
| 7346 | $(1 \wedge (c(x) \vee 0)) \vee (x \wedge 1) = 1$ | [279 → 6639,6384,5477,6384,18,12,18] |
| 7392 | $(x \vee 1) \wedge (x \vee ((0 \wedge (c(y) \vee 1)) \vee (y \wedge 0))) = x$ | [6785 → 43,6054,5477,6392] |
| 7680 | $c(0 \vee x) \vee x = 1$ | [5967 → 7346,6258] |
| 7694 | $c(x \vee 0) \vee (x \vee 0) = 1$ | [6499 → 7680] |
| 7744 | $(1 \wedge x) \vee c(x) = 1$ | [161 → 7680,6384,5477,5425,12,18] |
| 7809,7808 | $(x \vee x) \wedge 1 = x$ | [7680 → 1868,162,6384,5477,5425,12,18,46,5425,162,6384,5477,5425,12,18,46] |
| 7825 | $(0 \vee c(x \vee x)) \vee 1 = c(0 \vee (y \vee x)) \vee (y \vee 1)$ | [7680 → 261] |
| 7921 | $c(x \vee 0) \vee (x \vee 1) = 1$ | [7680 → 60,5653,6500] |
| 7950,7949 | $(0 \vee x) \vee 1 = x \vee 1$ | [7680 → 2397,2379,162,6384,5477,5425,12,18,4113] |
| 8056 | $c(x \vee x) \vee 1 = c(0 \vee (y \vee x)) \vee (y \vee 1)$ | [7825:7950] |

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| 8076,8075 | $(((x \wedge 1) \vee (x \wedge 0)) \wedge x) \vee 1 = x \vee 1$ | [7015:7950] |
| 8137 | $(x \vee x) \vee 1 = x \vee 1$ | [4229:7950] |
| 8142,8141 | $(1 \wedge x) \vee 1 = x \vee 1$ | [4112:7950] |
| 8153 | $(x \wedge x) \vee 1 = x \vee 1$ | [2378:7950] |
| 8208 | $(1 \wedge (c(x \vee x) \vee 0)) \vee x = 1$ | [7808 → 7346] |
| 8251 | $c(x \vee 0) = (c(x \vee x) \vee 0) \wedge 1$ | [7808 → 85,12] |
| 8261 | $(c(x \vee x) \vee 0) \wedge 1 = c(x \vee 0)$ | [8251] |
| 8289 | $x \vee c(x \vee x) = 1$ | [45 → 7744] |
| 8350 | $x \wedge (c(x) \vee c(x \vee 1)) = 0$ | [8289 → 201,12,162,28,6392,86,197,5477,42,7809,162,28,6392,8076] |
| 8354,8353 | $1 \wedge (x \vee ((x \vee y) \vee ((x \vee y) \vee y))) = x \vee y$ | [8289 → 449] |
| 8360 | $1 \wedge (x \vee (x \vee x)) = x$ | [8289 → 53] |
| 8566,8565 | $(x \vee 1) \vee (x \vee 1) = x \vee 1$ | [8137 → 501,8354] |
| 8728,8727 | $x \wedge (c(y) \vee c(y \vee 1)) = x \wedge (c(y) \vee c(y))$ | [8153 → 203,86,7809,30] |
| 8780 | $x \wedge (c(x) \vee c(x)) = 0$ | [8350:8728] |
| 8822 | $x \vee c(x \vee (x \vee x)) = 1$ | [8360 → 7744] |
| 8852 | $(c(x) \vee 1) \wedge ((x \wedge 0) \vee (x \wedge 0)) = 0$ | [6391 → 8780,6392] |
| 8878 | $(x \wedge 1) \vee (c(x) \vee 0) = 1$ | [279 → 7694,6384,5477,6384,18,12,18] |
| 8894 | $(x \vee c(c(x \vee 0) \vee (y \vee 0))) \wedge (y \vee 1) = x$ | [7694 → 449] |
| 9086 | $(x \wedge 1) \vee (c(x) \vee 1) = 1$ | [279 → 7921,6384,5477,6384,18,12,18] |
| 9090 | $c(x) \vee (x \vee 1) = 1$ | [8153 → 7921,86,7809] |
| 9111,9110 | $c(x \vee 1) \vee 1 = c(x \vee 0) \vee 1$ | [7921 → 1763,8566,8142] |
| 9200 | $c(0 \vee x) \vee (x \vee 1) = 1$ | [7949 → 9090] |
| 9222 | $x \vee (c(x) \vee 1) = 1$ | [2 → 9090] |
| 9226 | $(x \vee (x \wedge 0)) \wedge 1 = x$ | [9090 → 307,6438] |
| 9229,9228 | $c(x \vee 0) \vee 1 = c(x) \vee 1$ | [9090 → 1763,8566,8142,9111] |
| 9381,9380 | $(x \wedge 0) \vee 1 = x \vee 1$ | [9222 → 1763,8566,6392,8142] |
| 9398 | $(x \vee c(y \vee (z \vee (c(x) \vee 1)))) \wedge (y \vee (z \vee 1)) = x$ | [9222 → 131] |
| 9496 | $1 \wedge (c(x \vee (x \wedge 0)) \vee 0) = c(0 \vee x)$ | [9226 → 65,12] |
| 9517,9516 | $1 \vee ((x \wedge 0) \vee 0) = x \vee 1$ | [21 → 9380] |
| 9519,9518 | $c(x \vee 1) = (1 \vee (c(x) \vee 0)) \wedge 0$ | [6783:9517] |
| 9617,9616 | $(1 \vee (c(c(x)) \vee 0)) \wedge 0 = x \wedge 0$ | [6391:9519] |
| 10145 | $(x \vee (y \wedge (x \vee (x \vee x)))) \wedge (c(y) \vee 1) = x$ | [8822 → 393] |
| 10355 | $x \vee 1 = (y \wedge 1) \vee (c(y) \vee (x \vee 0))$ | [8878 → 16] |
| 10384 | $(x \wedge 1) \vee (c(x) \vee (y \vee 0)) = y \vee 1$ | [10355] |
| 10606 | $(x \wedge 1) \vee 1 = x \vee 1$ | [9086 → 1763,8566,9519,9617,8142,9381] |
| 10677,10676 | $c(x \vee x) \vee 1 = c(x) \vee 1$ | [663 → 10606,9229,7950] |
| 10715,10714 | $c(0 \vee (x \vee y)) \vee (x \vee 1) = c(y) \vee 1$ | [8056:10677] |
| 12016,12015 | $c(x) \vee 1 = 1$ | [8137 → 9200,10715] |
| 12036,12035 | $x \vee 1 = 1$ | [2 → 9200,12016] |
| 12057 | $(x \vee (y \wedge (x \vee (x \vee x)))) \wedge 1 = x$ | [10145:12036] |
| 12060,12059 | $(x \vee 0) \wedge 1 = x$ | [9398:12036,12036,12036,12,12036,12036] |
| 12068,12067 | $x \wedge 0 = 0$ | [8852:12036,46] |
| 12074,12073 | $1 \wedge (x \vee 0) = x$ | [7392:12036,12036,5914,12068,6384] |
| 12099 | $(x \vee (y \wedge x)) \wedge 1 = x$ | [391:12036] |
| 12163 | $(x \wedge 1) \vee (c(x) \vee (y \vee 0)) = 1$ | [10384:12036] |
| 12289 | $(x \vee c(c(x \vee 0) \vee (y \vee 0))) \wedge 1 = x$ | [8894:12036] |
| 12491 | $(x \vee c(y \vee (z \vee c(x)))) \wedge 1 = x$ | [1212:12036,12036] |
| 12519 | $(c(x) \vee 0) \vee (y \vee (x \wedge 1)) = 1$ | [138:12036] |
| 12525 | $c(x \vee x) = c(x \vee 0)$ | [8261:12060] |
| 12567 | $(x \vee c(y)) \wedge ((y \wedge 1) \vee (x \vee 0)) = x$ | [1878:12060] |
| 12608 | $c(x \vee 0) = c(0 \vee x)$ | [9496:12068,12074] |
| 12630 | $x \wedge 1 = 1 \wedge x$ | [7284:12068,12074] |
| 12681 | $c(x \vee x) \vee x = 1$ | [8208:12074] |
| 12762 | $1 \wedge x = x \wedge 1$ | [12630] |

| | | |
|---|---|---|
| 12928,12927 | $c(x \vee (y \wedge z)) = (c(x \vee 0) \vee 0) \wedge (c(y) \vee c(z))$ | [12059 → 196] |
| 12933 | $c((x \wedge 1) \vee y) = (c(x) \vee 0) \wedge (c(y \vee 0) \vee 0)$ | [12059 → 196,12] |
| 12959 | $(c(x) \vee 0) \wedge (c(y \vee 0) \vee 0) = c((x \wedge 1) \vee y)$ | [12933] |
| 12968 | $x \vee c(x \vee 0) = 1$ | [12073 → 7744] |
| 13051 | $1 \wedge (x \vee c((0 \vee c((x \vee y) \vee (x \vee y))) \vee y)) = x$ | [441 → 12630] |
| 13192 | $(x \vee (x \vee x)) \wedge 1 = x$ | [8360 → 12762] |
| 13198 | $(0 \vee (x \vee (x \vee x))) \wedge 1 = x$ | [4218 → 12762] |
| 13254 | $(x \vee (y \wedge 1)) \wedge (x \vee (0 \vee c(y))) = x$ | [12762 → 67] |
| 13480 | $(x \vee (((x \vee c(y)) \vee (x \vee c(y))) \wedge y)) \wedge 1 = x$ | [12681 → 393] |
| 13484 | $(x \vee (((y \wedge 1) \vee (y \wedge 1)) \wedge (c(y) \vee 0))) \wedge 1 = x$ | [12681 → 389,12036] |
| 13492 | $(x \vee c(c((x \vee y) \vee (x \vee y)) \vee (z \vee y))) \wedge 1 = x$ | [12681 → 449,12036] |
| 13645,13644 | $x \wedge (c(x) \vee 0) = 0$ | [12968 → 670,5863,12060] |
| 13708 | $(x \vee (x \wedge 1)) \wedge 1 = x$ | [12762 → 12099] |
| 13752 | $(c(x \vee 0) \vee 0) \wedge (c(y) \vee c(x)) = c(x \vee 0)$ | [12099 → 85,12928,12,12060] |
| 13762 | $x \vee ((0 \vee c((x \vee y) \vee 0)) \vee y) = 1$ | [12525 → 251] |
| 13960 | $c(0 \vee c(x)) = x \wedge 1$ | [200 → 12608,6384,5477,12036] |
| 14075,14074 | $(0 \vee x) \wedge 1 = x$ | [12608 → 27,28,12060] |
| 14120,14119 | $x \vee (x \vee x) = x$ | [13198:14075] |
| 14143,14142 | $x \wedge 1 = x$ | [13192:14120] |
| 14147,14146 | $x \vee (y \wedge x) = x$ | [12057:14120,14143] |
| 14149,14148 | $1 \wedge x = x$ | [8360:14120] |
| 14153,14152 | $0 \vee x = x$ | [4218:14120,14149] |
| 14183,14182 | $c(c(x)) = x$ | [13960:14153,14143] |
| 14229,14228 | $x \vee x = x$ | [13708:14143,14143] |
| 14256 | $x \vee c(c(x \vee y) \vee (z \vee y)) = x$ | [13492:14229,14143] |
| 14259,14258 | $x \vee 0 = x$ | [13484:14143,14143,14229,13645,14143] |
| 14260 | $x \vee ((x \vee c(y)) \wedge y) = x$ | [13480:14229,14143] |
| 14280 | $(x \vee y) \wedge (x \vee c(y)) = x$ | [13254:14143,14153] |
| 14293,14292 | $c(x \vee y) = c(x) \wedge c(y)$ | [12959:14259,14259,14259,14143] |
| 14294 | $(x \vee c(y)) \wedge (y \vee x) = x$ | [12567:14143,14259] |
| 14296 | $c(x) \vee (y \vee x) = 1$ | [12519:14259,14143] |
| 14300 | $x \vee (c(y) \wedge (c(z) \wedge x)) = x$ | [12491:14293,14293,14183,14143] |
| 14310 | $x \vee (x \wedge c(y)) = x$ | [12289:14259,14259,14293,14183,14143] |
| 14320 | $x \vee (c(x) \vee y) = 1$ | [12163:14143,14259] |
| 14350 | $(x \vee (c(y) \wedge z)) \wedge (y \vee (x \vee c(z))) = x$ | [3124:14259,14143] |
| 14352 | $(x \vee (c(y) \vee c(z))) \wedge ((y \wedge z) \vee x) = x$ | [666:14143,14259] |
| 14354 | $(x \vee y) \wedge (c(y) \vee x) = x$ | [425:14143,14259] |
| 14362 | $(x \vee c(y)) \vee (c(x) \wedge y) = 1$ | [235:14143,14259] |
| 14367,14366 | $c(x \wedge y) = c(x) \vee c(y)$ | [85:14259,14143] |
| 14370 | $x \vee ((c(x) \wedge c(y)) \vee y) = 1$ | [13762:14259,14293,14153] |
| 14374 | $c(x) \wedge (c(y) \vee c(x)) = c(x)$ | [13752:14259,14259,14259] |
| 14432 | $x \vee ((x \vee y) \wedge c(y)) = x$ | [13051:14229,14293,14153,14293,14367,14183,14183,14149] |
| 14465 | $x \vee ((x \vee y) \wedge (c(z) \wedge c(y))) = x$ | [14256:14293,14293,14367,14183,14183,14293] |
| 14528 | $x \vee (y \vee x) = y \vee x$ | [14228 → 2] |
| 14563 | $x \vee (y \vee z) = z \vee (y \vee x)$ | [14258 → 99,14259] |
| 14564 | $x \vee (y \vee z) = y \vee (z \vee x)$ | [14258 → 16,14259] |
| 14566 | $x \vee y = y \vee x$ | [14258 → 2,14259] |
| 14600,14599 | $x \vee (y \vee (z \wedge x)) = y \vee x$ | [14146 → 2] |
| 14602,14601 | $(x \wedge y) \vee y = y$ | [14146 → 14566] |
| 14738 | $A \vee (C \wedge (B \vee A)) \neq A \vee (B \wedge (A \vee C)) \mid \$Ans(MOD)$ | [14566 → 14] |
| 14739 | $(c(x) \vee y) \wedge (y \vee x) = y$ | [14566 → 7] |
| 14743 | $(c(x) \vee c(y)) \wedge (y \wedge x) = 0$ | [14566 → 37] |
| 14831 | $x \vee ((y \wedge (x \vee z)) \vee z) = x \vee z$ | [2 → 14601] |
| 14859 | $c(x) \wedge ((y \wedge c(x)) \vee x) = y \wedge c(x)$ | [14601 → 7] |

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|---|---|---|
| 14865 | $(x \vee y) \vee c(y) = 1$ | [14566 → 14296] |
| 14869 | $x \vee (x \wedge y) = x$ | [14182 → 14310] |
| 14882,14881 | $(x \wedge y) \vee x = x$ | [14566 → 14869] |
| 14909 | $x \vee (y \vee (x \wedge z)) = y \vee x$ | [14869 → 2] |
| 14967 | $(c(x) \vee y) \vee x = 1$ | [14566 → 14320] |
| 15143 | $(c(x) \vee y) \vee (z \vee x) = 1$ | [14967 → 2,12036] |
| 15283 | $x \vee ((c(y) \vee x) \wedge y) = x$ | [14566 → 14260] |
| 15302 | $x \vee ((c(x) \wedge y) \vee c(y)) = 1$ | [14260 → 14865,14367,14293,14183] |
| 15334 | $(x \vee y) \wedge (y \vee c(x)) = y$ | [14566 → 14280] |
| 15551 | $c(x) \wedge (x \vee (c(x) \wedge y)) = c(x) \wedge y$ | [14881 → 14294] |
| 15747 | $((x \vee y) \wedge c(y)) \vee x = x$ | [14566 → 14432] |
| 16076 | $(c(x) \vee y) \vee (x \vee z) = 1$ | [14566 → 15143] |
| 16203 | $x \vee (y \wedge (c(z) \wedge x)) = x$ | [14182 → 14300] |
| 16853 | $x \vee (y \wedge (z \wedge x)) = x$ | [14182 → 16203] |
| 16857 | $(x \wedge (y \wedge z)) \vee z = z$ | [14566 → 16853] |
| 16913,16912 | $(x \wedge y) \vee (y \vee z) = y \vee z$ | [14739 → 16857] |
| 17136 | $x \vee ((x \wedge c(y)) \wedge z) = x$ | [16076 → 14350,14293,14183,14143] |
| 17190 | $(x \wedge y) \vee (x \vee z) = x \vee z$ | [16076 → 14352,14183,14149,14183] |
| 17370 | $x \vee ((x \wedge y) \wedge z) = x$ | [14182 → 17136] |
| 17490 | $c(x) \vee (((x \wedge c(y)) \vee y) \wedge x) = 1$ | [15747 → 14362,14367,14293,14183,14183] |
| 17610 | $((x \wedge c(y)) \vee y) \vee x = (x \wedge c(y)) \vee y$ | [14370 → 15283,14183,14149,14183] |
| 17696 | $x \wedge (c(y) \vee x) = x$ | [14182 → 14374,14183,14183] |
| 17734 | $x \wedge (y \vee x) = x$ | [14182 → 17696] |
| 17786 | $(x \wedge y) \wedge y = x \wedge y$ | [14146 → 17734] |
| 18418,18417 | $(x \vee y) \vee z = y \vee (x \vee z)$ | [14566 → 14563] |
| 18571,18570 | $(x \wedge c(y)) \vee y = y \vee x$ | [17610:18418,14882] |
| 18928 | $c(x) \vee ((y \vee x) \wedge x) = 1$ | [17490:18571] |
| 18942 | $c(x) \wedge (x \vee y) = y \wedge c(x)$ | [14859:18571] |
| 18945 | $x \wedge c(y) = c(y) \wedge (y \vee x)$ | [18942] |
| 18989 | $x \vee (y \wedge c(x)) = x \vee y$ | [17786 → 18570,18571] |
| 19021 | $x \wedge (c(x) \vee y) = y \wedge x$ | [18570 → 15334,14183,14147,14183] |
| 19038,19037 | $(c(x) \vee y) \wedge x = y \wedge x$ | [18570 → 14294,14183,14147,14183] |
| 19047 | $x \wedge y = y \wedge (c(y) \vee x)$ | [19021] |
| 19087 | $(x \vee y) \wedge y = y$ | [18928 → 14354,14602,14143] |
| 19113 | $(x \vee y) \wedge x = x$ | [18570 → 19087] |
| 19122,19121 | $x \wedge (y \wedge (z \wedge x)) = y \wedge (z \wedge x)$ | [16853 → 19087] |
| 19257 | $c(x) \vee (y \wedge x) = c(x) \vee y$ | [14182 → 18989] |
| 19299 | $x \wedge (y \vee c(x)) = y \wedge x$ | [14566 → 19021] |
| 19303,19302 | $(x \vee c(y)) \wedge y = y \wedge (x \vee c(y))$ | [14528 → 19021] |
| 19324 | $x \wedge y = y \wedge (x \vee c(y))$ | [19299] |
| 19357 | $x \vee (y \wedge (x \vee c(y))) = x$ | [14260:19303] |
| 19592 | $(c(x) \wedge (x \vee y)) \vee x = x \vee y$ | [19047 → 18570,14183] |
| 19636 | $(x \wedge (c(x) \vee y)) \vee y = y$ | [19047 → 14881] |
| 19852 | $(x \wedge y) \vee (y \wedge (c(y) \vee x)) = x \wedge y$ | [18570 → 19357,14183,14183] |
| 20391,20390 | $(x \vee (y \wedge z)) \wedge (x \vee z) = x \vee (y \wedge z)$ | [14599 → 17734] |
| 20402 | $(x \vee (y \wedge c(z))) \wedge z = z \wedge (x \vee c(z))$ | [14599 → 19037,19303] |
| 20556 | $(x \vee (c(y) \wedge z)) \wedge y = y \wedge (x \vee c(y))$ | [14909 → 19037,19303] |
| 20672 | $x \vee ((x \wedge y) \vee c(y)) = (x \wedge y) \vee c(y)$ | [15302 → 19636,14143,14183,14183] |
| 20674 | $((x \wedge y) \vee c(y)) \wedge x = x$ | [15302 → 19037,14149,14183] |
| 20678 | $((x \wedge (y \vee c(x))) \vee c(x)) \wedge y = y$ | [19324 → 20674] |
| 20707,20706 | $x \vee (c(x) \wedge y) = x \vee y$ | [14280 → 20674,14293,14183,20391] |
| 20713,20712 | $c(x) \wedge (x \vee y) = c(x) \wedge y$ | [15551:20707] |
| 20718 | $(c(x) \wedge y) \vee x = x \vee y$ | [19592:20713] |
| 20720 | $x \wedge c(y) = c(y) \wedge x$ | [18945:20713] |

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| 20740 | $x \wedge y = y \wedge x$ | [14182 → 20720,14183] |
| 20742,20741 | $c(x) \wedge (y \vee x) = c(x) \wedge y$ | [19324 → 20720,14183] |
| 20768,20767 | $x \wedge (y \vee c(x)) = x \wedge y$ | [19324 → 20740] |
| 20770,20769 | $x \wedge (c(x) \vee y) = x \wedge y$ | [19047 → 20740] |
| 20775 | $((x \wedge y) \vee c(x)) \wedge y = y$ | [20678:20768] |
| 20778,20777 | $(x \vee (c(y) \wedge z)) \wedge y = y \wedge x$ | [20556:20768] |
| 20780,20779 | $(x \vee (y \wedge c(z))) \wedge z = z \wedge x$ | [20402:20768] |
| 20783 | $(x \wedge y) \vee (y \wedge x) = x \wedge y$ | [19852:20770] |
| 20960,20959 | $(x \vee c(y)) \wedge y = y \wedge x$ | [20740 → 20767] |
| 21072 | $(x \wedge y) \wedge (y \wedge x) = y \wedge x$ | [14743 → 20775,14293,14183,14183,14153] |
| 21087 | $x \vee ((y \wedge x) \vee c(y)) = (y \wedge x) \vee c(y)$ | [20775 → 14881] |
| 21465,21464 | $x \vee ((y \wedge x) \vee z) = x \vee z$ | [2 → 16912] |
| 21469,21468 | $(x \wedge y) \vee c(x) = y \vee c(x)$ | [21087:21465] |
| 21588,21587 | $x \vee ((x \wedge y) \vee z) = x \vee z$ | [2 → 17190] |
| 21594,21593 | $(x \wedge y) \vee c(y) = x \vee c(y)$ | [20672:21588] |
| 22207 | $(x \wedge c(y)) \vee (x \wedge y) = x$ | [20769 → 19257,14293,14183,14293,14183,14882] |
| 22209 | $(c(x) \wedge y) \vee (y \wedge x) = y$ | [20767 → 19257,14293,14183,14293,14183,14602] |
| 22240 | $(x \wedge c(y)) \vee (y \wedge x) = x$ | [20740 → 22207] |
| 22267 | $(x \wedge y) \vee (y \wedge c(x)) = y$ | [14182 → 22209] |
| 22271 | $(x \wedge y) \vee (c(y) \wedge x) = x$ | [14566 → 22209] |
| 22394 | $(x \wedge y) \vee (c(x) \wedge y) = y$ | [20740 → 22267] |
| 22462 | $x \vee ((x \vee y) \wedge (z \wedge c(y))) = x$ | [14182 → 14465] |
| 23045,23044 | $(x \wedge y) \vee ((y \wedge x) \vee z) = (y \wedge x) \vee z$ | [20783 → 18417] |
| 23048 | $x \vee (y \wedge z) = (z \wedge y) \vee x$ | [20783 → 14564,23045] |
| 23056 | $(x \wedge y) \vee z = z \vee (y \wedge x)$ | [23048] |
| 23273 | $(x \wedge y) \vee ((y \wedge x) \wedge z) = x \wedge y$ | [21072 → 17370] |
| 23304,23303 | $(x \vee (y \vee c(z))) \wedge (z \vee x) = (y \wedge z) \vee x$ | [21464 → 14294,18418,21594] |
| 23439,23438 | $((c(x) \wedge y) \vee z) \wedge x = z \wedge x$ | [21587 → 19037,19038] |
| 24842 | $x \wedge (y \wedge (x \vee c(z))) = y \wedge x$ | [18570 → 20777,23439,14367,14183] |
| 25351 | $x \vee ((y \vee x) \wedge (z \wedge c(y))) = x$ | [14566 → 22462] |
| 26810,26809 | $x \wedge (y \wedge (x \vee z)) = y \wedge x$ | [14182 → 24842] |
| 26841 | $(x \wedge y) \wedge (z \wedge x) = z \wedge (x \wedge y)$ | [22271 → 26809] |
| 26852 | $x \wedge (y \wedge (z \vee x)) = y \wedge x$ | [14909 → 26809] |
| 26870 | $x \wedge ((x \vee y) \wedge z) = x \wedge z$ | [20959 → 26809,14293,20778] |
| 26873 | $(x \wedge (y \vee z)) \wedge y = x \wedge y$ | [17786 → 26809,26810] |
| 26984,26983 | $(x \wedge (y \vee z)) \vee c(y) = x \vee c(y)$ | [26809 → 21468,21594] |
| 27027 | $x \wedge ((y \vee x) \wedge z) = x \wedge z$ | [20959 → 26852,14293,20780] |
| 27097 | $x \vee (y \wedge (z \vee c(x))) = x \vee y$ | [26852 → 20718,18571] |
| 27141 | $(x \wedge (y \vee z)) \vee c(z) = x \vee c(z)$ | [26852 → 21468,21594] |
| 27179,27178 | $(x \wedge y) \wedge (y \wedge z) = (x \wedge y) \wedge z$ | [22394 → 26870] |
| 27267 | $(((x \vee y) \wedge z) \wedge c(x)) \vee (x \wedge z) = (x \vee y) \wedge z$ | [26870 → 22240] |
| 27343,27342 | $(x \wedge y) \wedge z = x \wedge (y \wedge z)$ | [22271 → 26873,27179] |
| 27374 | $((x \vee y) \wedge (z \wedge c(x))) \vee (x \wedge z) = (x \vee y) \wedge z$ | [27267:27343] |
| 27453 | $x \wedge (y \wedge z) = y \wedge (z \wedge x)$ | [26841:27343,19122] |
| 27777 | $(x \wedge y) \vee (y \wedge (x \wedge z)) = x \wedge y$ | [23273:27343] |
| 28023,28022 | $((x \vee y) \wedge z) \vee c(y) = z \vee c(y)$ | [27027 → 21468,21469] |
| 28504,28503 | $(x \vee y) \wedge (z \wedge c(x)) = y \wedge (z \wedge c(x))$ | [25351 → 14294,28023,20960] |
| 28506 | $(x \wedge (y \wedge c(z))) \vee (z \wedge y) = (z \vee x) \wedge y$ | [27374:28504] |
| 28508 | $(x \vee y) \wedge z = (y \wedge (z \wedge c(x))) \vee (x \wedge z)$ | [28506] |
| 28889 | $((x \vee c(y)) \wedge z) \vee y = y \vee z$ | [23048 → 27097] |
| 28891 | $(x \wedge (y \vee c(z))) \vee z = z \vee x$ | [14566 → 27097] |
| 30251,30250 | $(x \wedge (y \wedge (z \vee c(u)))) \vee u = u \vee (x \wedge y)$ | [27453 → 28889] |
| 30648 | $(x \wedge (y \vee z)) \vee z = (x \wedge y) \vee z$ | [14831 → 14294,18418,26984,23304] |
| 32819 | $(x \vee c(y)) \wedge (y \vee (x \wedge (z \vee y))) = x \wedge (z \vee y)$ | [27141 → 14354,14183] |

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| 33046 | $(x \wedge ((x \wedge y) \vee z)) \vee (x \wedge y) = x \wedge ((x \wedge y) \vee z)$ | [19113 → 27777] |
| 33567,33566 | $(x \wedge (y \vee z)) \vee y = y \vee (x \wedge z)$ | [28891 → 30648,30251] |
| 33581,33580 | $(x \wedge (y \wedge c(z))) \vee (z \wedge y) = (x \wedge y) \vee (z \wedge y)$ | [22240 → 30648] |
| 33618,33617 | $x \vee (y \wedge (z \vee x)) = (y \wedge z) \vee x$ | [14566 → 30648] |
| 33632,33631 | $x \wedge ((x \wedge y) \vee z) = (x \wedge y) \vee (x \wedge z)$ | [33046:33567] |
| 33641,33640 | $(x \vee y) \wedge z = (y \wedge z) \vee (x \wedge z)$ | [28508:33581] |
| 33668,33667 | $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ | [32819:33618,33641,20742,33632,16913] |
| 33669 | $(C \wedge B) \vee A \neq A \vee (B \wedge C) \mid \$Ans(MOD)$ | [14738:33668,14600,33668,21465] |
| 33670 | $\$Ans(MOD)$ | [33669,23056] |

**Proof BA3**

| | | |
|---|---|---|
| 1 | $A \vee (A \wedge B) \neq A \mid \$Ans(B1)$ | [] |
| 2 | $c(c(A)) \neq A \mid \$Ans(CC)$ | [] |
| 4 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [] |
| 5 | $x \wedge y = c(c(x) \vee c(y))$ | [] |
| 6 | $c(c(x) \vee c(y)) = x \wedge y$ | [5] |
| 9 | $(x \vee c(y)) \wedge (x \vee y) = x$ | [] |
| 12,11 | $x \vee c(x) = 1$ | [] |
| 14,13 | $c(1) = 0$ | [] |
| 15 | $x \vee (y \vee (z \vee u)) = z \vee (x \vee (y \vee u))$ | [4 → 4] |
| 16 | $x \vee (y \vee (z \vee u)) = y \vee (z \vee (x \vee u))$ | [15] |
| 18,17 | $1 \vee 0 = 1$ | [13 → 11] |
| 19 | $x \vee (y \vee c(x)) = y \vee 1$ | [11 → 4] |
| 21 | $x \vee 1 = 1 \vee (x \vee 0)$ | [17 → 4] |
| 22 | $1 \vee (x \vee 0) = x \vee 1$ | [21] |
| 23 | $c(0 \vee c(x)) = 1 \wedge x$ | [13 → 6] |
| 25 | $c((x \wedge y) \vee c(z)) = (c(x) \vee c(y)) \wedge z$ | [6 → 6] |
| 28,27 | $c(c(x) \vee 0) = x \wedge 1$ | [13 → 6] |
| 29 | $c(c(x) \vee (y \wedge z)) = x \wedge (c(y) \vee c(z))$ | [6 → 6] |
| 31 | $x \wedge c(x) = 0$ | [11 → 6,14] |
| 34,33 | $(c(x) \vee c(y)) \vee (x \wedge y) = 1$ | [6 → 11] |
| 35 | $1 \wedge 0 = 0$ | [13 → 31] |
| 39 | $x \vee (1 \vee (y \vee 0)) = y \vee (x \vee 1)$ | [21 → 4] |
| 40 | $x \vee (y \vee 1) = y \vee (1 \vee (x \vee 0))$ | [39] |
| 42,41 | $(x \vee 0) \wedge (x \vee 1) = x$ | [13 → 9] |
| 43 | $(x \vee (y \wedge z)) \wedge (x \vee (c(y) \vee c(z))) = x$ | [6 → 9] |
| 46,45 | $1 \wedge (x \vee x) = x$ | [11 → 9] |
| 47 | $(x \vee 0) \wedge (1 \vee (x \vee 0)) = x$ | [21 → 9,14] |
| 49 | $(1 \vee c(0)) \wedge 1 = 1$ | [17 → 9] |
| 52,51 | $(x \vee c(c(x))) \wedge 1 = x$ | [11 → 9] |
| 53 | $(x \vee c(y \vee z)) \wedge (y \vee (x \vee z)) = x$ | [4 → 9] |
| 56,55 | $1 \wedge (x \vee ((x \vee y) \vee y)) = x \vee y$ | [4 → 45] |
| 59 | $x \vee (y \vee 1) = 1 \vee (x \vee (y \vee 0))$ | [22 → 4] |
| 60 | $1 \vee (x \vee (y \vee 0)) = x \vee (y \vee 1)$ | [59] |
| 62 | $x \vee (y \vee (1 \vee (z \vee 0))) = z \vee (x \vee (y \vee 1))$ | [21 → 15] |
| 64 | $x \vee (y \vee (z \vee c(x))) = y \vee (z \vee 1)$ | [11 → 15] |
| 68 | $x \vee (y \vee (z \vee u)) = z \vee (y \vee (x \vee u))$ | [4 → 15] |
| 70 | $x \vee (y \vee (z \vee 1)) = y \vee (z \vee (1 \vee (x \vee 0)))$ | [62] |
| 74 | $1 \wedge (x \vee ((y \vee (x \vee z)) \vee (y \vee z))) = y \vee (x \vee z)$ | [15 → 45] |
| 76 | $(x \vee c(y \vee (z \vee u))) \wedge (z \vee (x \vee (y \vee u))) = x$ | [15 → 9] |
| 83,82 | $c(0 \vee 0) = 1 \wedge 1$ | [13 → 23] |

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| 84 | $c(0 \vee (x \wedge y)) = 1 \wedge (c(x) \vee c(y))$ | [6 → 23] |
| 86 | $(x \vee (1 \wedge y)) \wedge (x \vee (0 \vee c(y))) = x$ | [23 → 9] |
| 90 | $(0 \vee c(x)) \vee (1 \wedge x) = 1$ | [23 → 11] |
| 92 | $(x \vee (1 \wedge 1)) \wedge (x \vee (0 \vee 0)) = x$ | [82 → 9] |
| 96 | $(0 \vee 0) \vee (1 \wedge 1) = 1$ | [82 → 11] |
| 105,104 | $c((x \wedge y) \vee 0) = (c(x) \vee c(y)) \wedge 1$ | [6 → 27] |
| 108 | $(c(x) \vee 0) \wedge (x \wedge 1) = 0$ | [27 → 31] |
| 110 | $(c(x) \vee 0) \vee (x \wedge 1) = 1$ | [27 → 11] |
| 116 | $x \vee (y \vee 1) = 1 \vee (y \vee (x \vee 0))$ | [22 → 16] |
| 124 | $1 \vee (x \vee (y \vee 0)) = y \vee (x \vee 1)$ | [116] |
| 126 | $1 \wedge (x \vee (y \vee ((x \vee (y \vee z)) \vee z))) = x \vee (y \vee z)$ | [16 → 45] |
| 130 | $(x \vee c(y \vee (z \vee u))) \wedge (y \vee (z \vee (x \vee u))) = x$ | [16 → 9] |
| 133 | $((0 \vee 0) \vee c(1 \wedge 1)) \wedge 1 = 0 \vee 0$ | [82 → 51] |
| 167 | $c(x \vee c(y)) = (0 \vee c(x \vee x)) \wedge y$ | [45 → 25,14] |
| 169,168 | $c(x \vee c(y)) = (c(x \vee 0) \vee c(x \vee 1)) \wedge y$ | [41 → 25] |
| 171,170 | $((1 \wedge 1) \vee c(0 \vee 1)) \wedge x = 1 \wedge x$ | [35 → 25,169,83,14,12] |
| 181,180 | $c((x \wedge y) \vee (z \wedge u)) = (c(x) \vee c(y)) \wedge (c(z) \vee c(u))$ | [6 → 25] |
| 184 | $(0 \vee c(x \vee x)) \wedge y = (c(x \vee 0) \vee c(x \vee 1)) \wedge y$ | [167,169] |
| 190 | $((x \wedge 1) \vee c(c(x) \vee 1)) \wedge y = x \wedge y$ | [6:169,28] |
| 192 | $(0 \vee (1 \wedge 1)) \vee 0 = 1$ | [82 → 90,46] |
| 196 | $(0 \vee c(x \vee x)) \vee x = 1$ | [45 → 90] |
| 200 | $1 \wedge ((0 \vee (1 \wedge 1)) \vee 1) = 0 \vee (1 \wedge 1)$ | [192 → 41] |
| 203,202 | $(0 \vee (1 \wedge 1)) \vee 1 = 1 \vee 1$ | [192 → 22] |
| 209 | $0 \vee (1 \wedge 1) = 1$ | [200:203,46] |
| 214 | $(0 \vee c(1 \wedge 1)) \wedge 1 = 0$ | [209 → 9] |
| 216 | $0 \vee (x \vee (1 \wedge 1)) = x \vee 1$ | [209 → 4] |
| 218 | $c(c(x) \vee y) = x \wedge (((c(y \vee 0) \vee c(y \vee 1)) \wedge c(y)) \vee 0)$ | [51 → 29,169,14] |
| 220,219 | $c(c(x) \vee 1) = x \wedge (((0 \vee c(1 \vee 1)) \wedge 0) \vee 0)$ | [49 → 29,169,18,14,14] |
| 221 | $c(c(x) \vee y) = x \wedge (0 \vee c(y \vee y))$ | [45 → 29,14] |
| 222 | $c(c(x) \vee y) = x \wedge (c(y \vee 0) \vee c(y \vee 1))$ | [41 → 29] |
| 224 | $x \wedge (((c(y \vee 0) \vee c(y \vee 1)) \wedge c(y)) \vee 0) = c(c(x) \vee y)$ | [218] |
| 226,225 | $((x \wedge 1) \vee (x \wedge (((0 \vee c(1 \vee 1)) \wedge 0) \vee 0))) \wedge y = x \wedge y$ | [190:220] |
| 227 | $x \wedge (0 \vee c(y \vee y)) = c(c(x) \vee y)$ | [221] |
| 242 | $(c(x) \vee (y \wedge z)) \vee (x \wedge (c(y) \vee c(z))) = 1$ | [29 → 11] |
| 248 | $((x \wedge 1) \vee 0) \wedge ((c(x) \vee 0) \wedge 1) = 0$ | [27 → 108] |
| 258 | $(((c(x \vee 0) \vee c(x \vee 1)) \wedge c(x)) \vee 0) \vee x = 1$ | [51 → 110,169] |
| 282 | $(c(x \vee 0) \vee c(x \vee 1)) \vee x = 1$ | [41 → 33] |
| 286 | $(c(x) \vee c(y)) \vee (z \vee (u \vee (x \wedge y))) = z \vee (u \vee 1)$ | [33 → 15] |
| 290 | $(0 \vee c(x \vee ((y \vee (x \vee z)) \vee (y \vee z)))) \vee (y \vee (x \vee z)) = 1$ | [15 → 196] |
| 305,304 | $(0 \vee (x \wedge x)) \vee 1 = x \vee 1$ | [196 → 19,169,28,220,226] |
| 307 | $x \vee (y \vee ((0 \vee c((x \vee z) \vee (x \vee z))) \vee z)) = y \vee 1$ | [196 → 15] |
| 315 | $x \vee 1 = (0 \vee c(y \vee y)) \vee (x \vee y)$ | [196 → 4] |
| 319 | $(0 \vee c(x \vee x)) \vee (y \vee x) = y \vee 1$ | [315] |
| 321,320 | $((1 \wedge (1 \wedge 1)) \vee 0) \vee 0 = 1$ | [214 → 110,169,83,171] |
| 367,366 | $(0 \vee 0) \vee 1 = 0 \vee 1$ | [196 → 216,181,14,14,14,14,305] |
| 386 | $x \vee (y \vee 1) = x \vee (1 \vee (y \vee 0))$ | [4 → 40] |
| 420,419 | $(1 \wedge (1 \wedge 1)) \vee 0 = 1$ | [320 → 47,321,46] |
| 422 | $1 \wedge (1 \wedge 1) = 1$ | [419 → 47,420,46] |
| 430 | $(x \vee y) \wedge (x \vee (0 \vee c(y \vee y))) = x$ | [45 → 43,14] |
| 438 | $1 \wedge (0 \vee (0 \vee 0)) = 0$ | [209 → 43,14,14] |
| 452 | $(x \vee (y \wedge x)) \wedge (c(y) \vee 1) = x$ | [19 → 43] |
| 454 | $(x \vee (y \wedge z)) \wedge (c(y) \vee (x \vee c(z))) = x$ | [4 → 43] |
| 464 | $(x \vee 1) \wedge (x \vee (0 \vee c(1 \wedge 1))) = x$ | [422 → 43,14] |
| 466 | $x \wedge (0 \vee c(1 \wedge 1)) = x \wedge (((0 \vee c(1 \vee 1)) \wedge 0) \vee 0)$ | [422 → 29,220,14] |

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| 469,468 | $(0 \vee c(0 \vee (0 \vee 0))) \vee 0 = 1$ | [438 → 90] |
| 503 | $(x \vee (1 \wedge 1)) \wedge (0 \vee (x \vee 0)) = x$ | [82 → 53] |
| 639 | $((1 \wedge 1) \vee 1) \wedge (0 \vee 1) = 1 \wedge 1$ | [422 → 452,14] |
| 651 | $((x \vee x) \vee x) \wedge (0 \vee 1) = x \vee x$ | [45 → 452,14] |
| 714 | $0 \vee c(0 \vee (0 \vee 0)) = 1$ | [468 → 47,469,46] |
| 744,743 | $(0 \vee c((x \vee 0) \vee (x \vee 0))) \vee (x \vee 1) = 1 \vee 1$ | [196 → 60] |
| 903,902 | $((1 \wedge 1) \vee c(0 \vee 1)) \vee 0 = 1$ | [82 → 282] |
| 946 | $(1 \wedge 1) \vee c(0 \vee 1) = 1$ | [282 → 47,83,903,46,83] |
| 1015 | $(1 \wedge 1) \vee c(1 \vee (0 \vee 0)) = 1$ | [21 → 946] |
| 1017 | $1 \wedge (0 \vee ((1 \wedge 1) \vee 1)) = 1 \wedge 1$ | [946 → 53] |
| 1020,1019 | $(1 \wedge 1) \vee 1 = (0 \vee 1) \vee 1$ | [946 → 19] |
| 1037,1036 | $1 \wedge 1 = 0 \vee 1$ | [1017:1020,56] |
| 1043,1042 | $((0 \vee 1) \vee 1) \wedge (0 \vee 1) = 0 \vee 1$ | [639:1037,1037] |
| 1044 | $(0 \vee 1) \vee c(1 \vee (0 \vee 0)) = 1$ | [1015:1037] |
| 1092 | $(x \vee (0 \vee 1)) \wedge (0 \vee (x \vee 0)) = x$ | [503:1037] |
| 1105,1104 | $x \wedge (((0 \vee c(1 \vee 1)) \wedge 0) \vee 0) = x \wedge (0 \vee c(0 \vee 1))$ | [466:1037] |
| 1106 | $(x \vee 1) \wedge (x \vee (0 \vee c(0 \vee 1))) = x$ | [464:1037] |
| 1139,1138 | $0 \vee (0 \vee 1) = 1$ | [209:1037] |
| 1144 | $((0 \vee 0) \vee c(0 \vee 1)) \wedge 1 = 0 \vee 0$ | [133:1037] |
| 1146 | $(0 \vee 0) \vee (0 \vee 1) = 1$ | [96:1037] |
| 1148 | $(x \vee (0 \vee 1)) \wedge (x \vee (0 \vee 0)) = x$ | [92:1037] |
| 1151,1150 | $c(0 \vee 0) = 0 \vee 1$ | [82:1037] |
| 1161,1160 | $((x \wedge 1) \vee (x \wedge (0 \vee c(0 \vee 1)))) \wedge y = x \wedge y$ | [225:1105] |
| 1163,1162 | $c(c(x) \vee 1) = x \wedge (0 \vee c(0 \vee 1))$ | [219:1105] |
| 1164 | $(1 \vee (0 \vee 1)) \wedge (0 \vee 1) = 1$ | [1036 → 452,14] |
| 1166 | $c(c(x) \vee (0 \vee 1)) = x \wedge (0 \vee 0)$ | [1036 → 29,14,14] |
| 1181 | $0 \vee (1 \vee (0 \vee 0)) = 1$ | [21 → 1138] |
| 1190,1189 | $0 \vee (0 \vee (x \vee 1)) = x \vee 1$ | [1138 → 16] |
| 1203 | $(0 \vee 0) \vee (0 \vee (x \vee 1)) = x \vee 1$ | [1146 → 16] |
| 1286 | $0 \vee (1 \vee (x \vee (0 \vee 0))) = x \vee 1$ | [1181 → 16] |
| 1295 | $0 \vee (x \vee (1 \vee (0 \vee 0))) = x \vee 1$ | [1181 → 4] |
| 1459,1458 | $(0 \vee (1 \vee 1)) \wedge (0 \vee 1) = 1$ | [40 → 1164,18] |
| 1550,1549 | $x \wedge (c(0 \vee (1 \vee 1)) \vee c(0 \vee 1)) = x \wedge (0 \vee c(0 \vee 1))$ | [1458 → 29,1163] |
| 1560,1559 | $(1 \vee (0 \vee 0)) \vee 1 = (0 \vee 1) \vee 1$ | [1044 → 19] |
| 1896 | $(x \vee (1 \vee (((x \vee 1) \vee (x \vee 1)) \vee 0))) \wedge (0 \vee 1) = (x \vee 1) \vee (x \vee 1)$ | [40 → 651] |
| 2005 | $(c((0 \vee 1) \vee 1) \vee c(0 \vee 1)) \vee (0 \vee 1) = 1$ | [1042 → 33] |
| 2135 | $(0 \vee (x \vee 1)) \wedge (0 \vee (x \vee 0)) = x$ | [4 → 1092] |
| 2213 | $(0 \vee (x \vee 1)) \wedge (x \vee (0 \vee 0)) = x$ | [4 → 1148] |
| 2216,2215 | $0 \vee c((0 \vee 0) \vee (0 \vee 0)) = (1 \vee 1) \wedge 1$ | [196 → 1148,744] |
| 2235 | $c(0 \vee (c(x) \vee 1)) = x \wedge (0 \vee 0)$ | [4 → 1166] |
| 2361,2360 | $(c((0 \vee 0) \vee 0) \vee c(0 \vee 1)) \vee 1 = 0 \vee (1 \vee 1)$ | [282 → 1286,367] |
| 2384,2383 | $1 \wedge (0 \vee c(0 \vee 1)) = c(0 \vee 1)$ | [1458 → 84,1550] |
| 2405 | $c(0 \vee x) = 1 \wedge (0 \vee c(x \vee x))$ | [45 → 84,14] |
| 2411 | $1 \wedge (0 \vee c(x \vee x)) = c(0 \vee x)$ | [2405] |
| 2535 | $(x \vee (1 \wedge y)) \wedge (0 \vee (x \vee c(y))) = x$ | [4 → 86] |
| 2733 | $1 \wedge ((1 \vee (0 \vee 0)) \vee (((0 \vee 1) \vee 1) \vee 1)) = (0 \vee 1) \vee 1$ | [1559 → 55,1560] |
| 2816,2815 | $c((0 \vee 0) \vee 0) \vee c(0 \vee 1) = (1 \vee 1) \wedge 1$ | [282 → 2213,367,2361,1190,367] |
| 3021 | $c(1 \vee (0 \vee 0)) = 1 \wedge (0 \vee c(1 \vee 1))$ | [21 → 2405] |
| 3107 | $0 \vee (1 \wedge ((1 \vee 1) \wedge 1)) = 1$ | [2405 → 714,2216] |
| 3185 | $1 \wedge (0 \vee c((1 \vee 1) \wedge 1)) = 0$ | [3107 → 84,14,14] |
| 3208 | $0 \vee (x \vee (1 \wedge ((1 \vee 1) \wedge 1))) = x \vee 1$ | [3107 → 4] |
| 3210 | $((0 \vee c((1 \vee 1) \wedge 1)) \vee 0) \wedge (0 \vee 1) = 0 \vee c((1 \vee 1) \wedge 1)$ | [3185 → 452,14] |
| 3865 | $x \wedge (0 \vee c(y \vee ((1 \vee (y \vee 0)) \vee 1))) = c(c(x) \vee (1 \vee (y \vee 0)))$ | [39 → 227] |
| 3867,3866 | $x \wedge (0 \vee c(1 \vee 1)) = x \wedge (0 \vee c(0 \vee 1))$ | [21 → 227,18,1163] |

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| 3877 | $c(c(x) \vee (1 \vee (y \vee 0))) = x \wedge (0 \vee c(y \vee ((1 \vee (y \vee 0)) \vee 1)))$ | [3865] |
| 3881,3880 | $c(1 \vee (0 \vee 0)) = c(0 \vee 1)$ | [3021:3867,2384] |
| 4923,4922 | $(0 \vee (x \wedge x)) \vee (y \vee 1) = x \vee (y \vee 1)$ | [319 → 64,169,28,1163,1161] |
| 5028 | $(x \vee (1 \vee (y \vee 0))) \wedge (x \vee (0 \vee c(y \vee ((1 \vee (y \vee 0)) \vee 1)))) = x$ | [39 → 430] |
| 5587 | $0 \vee ((0 \vee 1) \wedge (0 \vee 1)) = 0 \vee 1$ | [319 → 1106,169,28,1163,1161,305,1043,169,28,1163,1161] |
| 5612,5611 | $(c(0 \vee 1) \vee c(0 \vee 1)) \vee (0 \vee 1) = 0 \vee 1$ | [5587 → 319,181,4923] |
| 5840 | $((0 \vee 0) \vee 0) \wedge (((((1 \vee 1) \wedge 1) \wedge (0 \vee 1)) \vee 0) \wedge 1) = 0$ | [1144 → 248,169,367,2816] |
| 5844 | $c((0 \vee 0) \vee 0) = ((((1 \vee 1) \wedge 1) \wedge (0 \vee 1)) \vee 0) \wedge 1$ | [1144 → 104,169,367,2816,14] |
| 5866,5865 | $c((0 \vee 1) \vee 1) = (0 \vee 0) \wedge (0 \vee c(0 \vee 1))$ | [1150 → 1162] |
| 5883 | $(((0 \vee 0) \wedge (0 \vee c(0 \vee 1))) \vee c(0 \vee 1)) \vee (0 \vee 1) = 1$ | [2005:5866] |
| 6410 | $c(x \vee (0 \vee 1)) = (c(x \vee 0) \vee c(x \vee 1)) \wedge (0 \vee 0)$ | [1150 → 168] |
| 6422 | $x \wedge (0 \vee (y \wedge y)) = x \wedge y$ | [221 → 168,169,28,1163,1161,28,1163,1161] |
| 7247 | $x \wedge ((y \wedge c(c(y))) \vee 0) = x \wedge y$ | [168 → 218,28,1163,1161,28,1163,1161] |
| 7474 | $c((x \wedge (0 \vee 0)) \vee y) = (0 \vee (c(x) \vee 1)) \wedge (c(y \vee 0) \vee c(y \vee 1))$ | [2235 → 222] |
| 9519 | $((x \wedge c(y)) \vee (x \wedge y)) \vee c(x) = 1$ | [9 → 242,169,28,1163,1161] |
| 9633 | $((1 \wedge x) \vee c(0 \vee x)) \vee 0 = 1$ | [2411 → 9519,169,1151,12,46,14] |
| 10131,10130 | $((1 \wedge x) \vee c(0 \vee x)) \vee 1 = 1 \vee 1$ | [9633 → 319,1151,1139] |
| 10150 | $(1 \wedge x) \vee c(0 \vee x) = 1$ | [9633 → 2135,10131,1459] |
| 10186 | $(1 \wedge x) \vee c(0 \vee ((x \wedge c(c(x))) \vee 0)) = 1$ | [7247 → 10150] |
| 10212 | $c(0 \vee x) \vee c(0 \vee (((c(x \vee 0) \vee c(x \vee 1)) \wedge c(x)) \vee 0)) = 1$ | [224 → 10150,14] |
| 10220 | $x \vee c(0 \vee (x \vee x)) = 1$ | [45 → 10150] |
| 10301,10300 | $(1 \wedge x) \vee 1 = (0 \vee x) \vee 1$ | [10150 → 319,169,28,1163,1161,305] |
| 10440 | $(0 \vee (x \vee x)) \vee 1 = x \vee 1$ | [10220 → 319,169,28,1163,1161,305] |
| 10444,10443 | $1 \wedge (0 \vee (x \vee (x \vee x))) = x$ | [10220 → 130] |
| 10445 | $1 \wedge (x \vee (x \vee (0 \vee x))) = x$ | [10220 → 76] |
| 10688 | $(0 \vee ((x \wedge c(c(x))) \vee 0)) \vee 1 = (0 \vee x) \vee 1$ | [7247 → 10300,10301] |
| 10690 | $(0 \vee (0 \vee (x \wedge x))) \vee 1 = (0 \vee x) \vee 1$ | [6422 → 10300,10301] |
| 10848,10847 | $((0 \vee 1) \vee 1) \vee 1 = (0 \vee 1) \vee 1$ | [1295 → 10440,1560,1560] |
| 10902,10901 | $1 \wedge ((1 \vee (0 \vee 0)) \vee ((0 \vee 1) \vee 1)) = (0 \vee 1) \vee 1$ | [2733:10848] |
| 10903 | $0 \vee (1 \vee 1) = 1$ | [10440 → 126,10444] |
| 11101,11100 | $((x \wedge c(c(x))) \vee 0) \vee 1 = x \vee 1$ | [258 → 19,28,1163,1161] |
| 11131,11130 | $1 \vee 1 = 0 \vee 1$ | [10903 → 1203,367] |
| 11458 | $c((0 \vee 0) \vee 0) = ((((0 \vee 1) \wedge 1) \wedge (0 \vee 1)) \vee 0) \wedge 1$ | [5844:11131] |
| 11460 | $((0 \vee 0) \vee 0) \wedge (((((0 \vee 1) \wedge 1) \wedge (0 \vee 1)) \vee 0) \wedge 1) = 0$ | [5840:11131] |
| 11596 | $((0 \vee c((0 \vee 1) \wedge 1)) \vee 0) \wedge (0 \vee 1) = 0 \vee c((0 \vee 1) \wedge 1)$ | [3210:11131,11131] |
| 11598 | $0 \vee (x \vee (1 \wedge ((0 \vee 1) \wedge 1))) = x \vee 1$ | [3208:11131] |
| 11992,11991 | $(0 \vee 1) \vee 1 = 1 \vee (0 \vee 0)$ | [1181 → 10445,1560,10902] |
| 12052,12051 | $(0 \vee 0) \wedge (0 \vee c(0 \vee 1)) = c(0 \vee 1)$ | [5865:11992,3881] |
| 12140,12139 | $0 \vee 1 = 1$ | [5883:12052,5612] |
| 12253,12252 | $0 \vee (x \vee 1) = x \vee 1$ | [11598:12140,1037,12140,1037,12140] |
| 12255,12254 | $((0 \vee 0) \vee 0) \wedge 1 = 0 \vee 0$ | [11596:12140,1037,12140,14,12140,12140,1037,12140,14] |
| 12354,12353 | $0 \vee 0 = 0$ | [11460:12140,1037,12140,12140,1037,12140,18,1037,12140,12255] |
| 12356,12355 | $c(0) = 1$ | [11458:12354,12354,12140,1037,12140,12140,1037,12140,18,1037,12140] |
| 12406,12405 | $1 \vee 1 = 1$ | [11130:12140] |
| 12584,12583 | $(c(x \vee 0) \vee c(x \vee 1)) \wedge 0 = c(x \vee 1)$ | [6410:12140,12354] |
| 12810 | $(x \vee (1 \vee (((x \vee 1) \vee (x \vee 1)) \vee 0))) \wedge 1 = (x \vee 1) \vee (x \vee 1)$ | [1896:12140] |
| 12827,12826 | $c(c(x) \vee 1) = x \wedge 0$ | [1162:12140,14,12354] |
| 12829,12828 | $((x \wedge 1) \vee (x \wedge 0)) \wedge y = x \wedge y$ | [1160:12140,14,12354] |
| 12896 | $c((x \wedge 0) \vee y) = (c(x) \vee 1) \wedge (c(y \vee 0) \vee c(y \vee 1))$ | [7474:12354,12253] |
| 13249,13248 | $0 \vee (x \vee (y \vee 0)) = x \vee (y \vee 0)$ | [12353 → 15] |
| 13257,13256 | $0 \vee (x \vee 0) = x \vee 0$ | [12353 → 4] |
| 13315,13314 | $(0 \vee x) \vee 1 = x \vee 1$ | [10688:13257,11101] |
| 13316 | $c(0 \vee x) \vee x = 1$ | [10212:13257,105,169,28,12827,12829,42,52] |
| 13318 | $(1 \wedge x) \vee c(x) = 1$ | [10186:13257,105,52] |

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| 13448 | $(x \wedge x) \vee 1 = x \vee 1$ | [10690:13315,13315,13315] |
| 13452 | $(x \vee x) \vee 1 = x \vee 1$ | [10440:13315] |
| 13859 | $x \vee (c(x \vee 0) \vee 1) = 1$ | [13316 → 124,12406,13257] |
| 13861 | $c(x \vee 0) \vee (x \vee 1) = 1$ | [13316 → 60,12406,13257] |
| 14031 | $x \vee c(x \vee x) = 1$ | [45 → 13318] |
| 14186,14185 | $(x \vee x) \wedge 1 = x$ | [14031 → 2535,46,12140] |
| 14305,14304 | $(x \vee ((x \vee y) \vee y)) \wedge 1 = x \vee y$ | [4 → 14185] |
| 14332 | $(c(x \vee x) \vee 0) \vee x = 1$ | [14185 → 110] |
| 14719 | $c(x \vee 0) \vee (x \vee 0) = 1$ | [13256 → 13316] |
| 14843 | $(1 \vee (x \vee 0)) \vee 1 = (x \vee 1) \vee 1$ | [386 → 13314,13249] |
| 14882,14881 | $1 \vee ((0 \vee x) \vee 0) = x \vee 1$ | [21 → 13314] |
| 15156,15155 | $1 \vee ((x \vee x) \vee 0) = x \vee 1$ | [21 → 13452] |
| 15158,15157 | $(x \vee 1) \vee (x \vee 1) = x \vee 1$ | [12810:15156,14305] |
| 15355,15354 | $1 \wedge (x \vee 1) = x \vee 1$ | [286 → 74,34,15158,34] |
| 15822 | $x \vee (y \vee (c(y \vee ((x \vee (y \vee 1)) \vee (x \vee 1))) \vee 1)) = 1$ | [70 → 290,14882] |
| 15828 | $x \vee (c(x \vee ((1 \vee (x \vee 0)) \vee 1)) \vee 1) = 1$ | [39 → 290,18,13315] |
| 15842 | $x \vee (y \vee (c(x \vee ((1 \vee (x \vee 0)) \vee 1)) \vee 1)) = y \vee 1$ | [290 → 62,18,13315] |
| 16165 | $(x \vee c(y \vee (z \vee (c(x \vee 0) \vee 1)))) \wedge (y \vee (z \vee 1)) = x$ | [13859 → 130] |
| 16209 | $c(x) \vee (x \vee 1) = 1$ | [13448 → 13861,105,14186] |
| 16220,16219 | $c(x \vee 1) \vee 1 = c(x \vee 0) \vee 1$ | [13861 → 319,15158,13315] |
| 16363 | $x \vee (c(x) \vee 1) = 1$ | [4 → 16209] |
| 16366,16365 | $c(x \vee 0) \vee 1 = c(x) \vee 1$ | [16209 → 319,15158,13315,16220] |
| 16466,16465 | $c(x \vee 1) \vee 1 = c(x) \vee 1$ | [16219:16366] |
| 16469 | $(x \vee c(y \vee (z \vee (c(x) \vee 1)))) \wedge (y \vee (z \vee 1)) = x$ | [16165:16366] |
| 16500 | $(x \wedge 0) \vee 1 = x \vee 1$ | [16363 → 319,15158,12827,13315] |
| 16723,16722 | $c(x \vee x) \vee 1 = c(x) \vee 1$ | [184 → 16500,12584,16466,13315] |
| 17210 | $(c(x \vee ((1 \vee (x \vee 0)) \vee 1)) \vee 0) \vee (1 \vee (x \vee 0)) = 1$ | [39 → 14332] |
| 17536 | $x \vee 1 = y \vee (c(y \vee 0) \vee (x \vee 0))$ | [14719 → 68] |
| 17573 | $x \vee (c(x \vee 0) \vee (y \vee 0)) = y \vee 1$ | [17536] |
| 17794,17793 | $(x \vee 1) \vee 1 = x \vee 1$ | [16209 → 307,15158,169,12354,12356,12140,14,18,15355] |
| 17796,17795 | $x \vee (c(x \vee y) \vee 1) = c(y) \vee 1$ | [319 → 307,13315,16723,13315,16723] |
| 17875,17874 | $(1 \vee (x \vee 0)) \vee 1 = x \vee 1$ | [14843:17794] |
| 17905,17904 | $c(x) \vee 1 = 1$ | [15828:17875,17796,16466] |
| 17907,17906 | $x \vee (y \vee 1) = 1$ | [15822:17905] |
| 17916,17915 | $1 \vee (x \vee 0) = 1$ | [17210:17875,17907,14,12354,13249] |
| 17918,17917 | $x \vee 1 = 1$ | [15842:17916,12406,17905,17907] |
| 17920,17919 | $1 \wedge (x \vee 0) = x$ | [5028:17916,17918,17916,17918,17918,14,12354] |
| 17922,17921 | $x \wedge 0 = 0$ | [3877:17916,17918,14,17916,17918,17918,14,12354] |
| 17980,17979 | $(x \vee 0) \wedge 1 = x$ | [16469:17918,17918,17918,14,17918,17918] |
| 17989 | $c(0 \vee x) = c(x \vee 0)$ | [12896:17922,17918,17918,14,17920] |
| 18409 | $x \vee (c(x \vee 0) \vee (y \vee 0)) = 1$ | [17573:17918] |
| 20575 | $(0 \vee x) \wedge 1 = x$ | [17989 → 27,28,17980] |
| 20873,20872 | $x \vee 0 = x$ | [13256 → 20575,17980] |
| 21639 | $x \vee (c(x) \vee y) = 1$ | [18409:20873,20873] |
| 21688,21687 | $x \wedge 1 = x$ | [17979:20873] |
| 21862,21861 | $c(c(x)) = x$ | [27:20873,21688] |
| 21863 | $\$Ans(CC)$ | [21861,2] |
| 22975 | $x \vee (x \wedge y) = x$ | [21639 → 454,21862,21688,21862] |
| 22977 | $\$Ans(B1)$ | [22975,1] |

Independence of the $\mathcal{BA}$ 4-basis {AJ, DM, ONE, CUT} is open. In particular, we have not been able to find a proof or countermodel of

{AJ, DM, CUT} ⇒ ONE.

The simplest multiequation basis we know of for $\mathcal{BA}$ in terms of join and complement is the following, due to C. A. Meredith [13].

$c(c(x) \vee y) \vee x = x$ % MER_1
$c(c(x) \vee y) \vee (z \vee y) = y \vee (z \vee x)$ % MER_2

For comparison, the Robbins 3-basis for $\mathcal{BA}$ (in terms of join and complement) is the following [12].

$(x \vee y) \vee z = x \vee (y \vee z)$ % AJ2
$x \vee y = y \vee x$ % CJ
$c(c(x \vee c(y)) \vee c(x \vee y)) = x$ % Robbins

## 2.2 In Terms of the Sheffer Stroke

This section contain multiequation bases, in terms of the Sheffer stroke, for $\mathcal{OL}$, $\mathcal{OML}$, $\mathcal{MOL}$, and $\mathcal{BA}$. (There cannot be a basis for $\mathcal{L}$ in terms of the Sheffer stroke.) We prove each basis equivalent to the corresponding basis in terms of join, meet, and complement with two Otter jobs, and we show independence with Mace2 countermodels.

### 2.2.1 Ortholattices in Terms of the Sheffer Stroke

The following is a 3-basis for ortholattices in terms of the Sheffer stroke.

$(x|((y|z)|(y|z))) = (y|((x|z)|(x|z)))$ % A_SS
$((x|x)|(x|y)) = x$ % B_SS
$(x|(x|x)) = (y|(y|y))$ % ONE_SS

The following two Otter jobs show that this basis is definitionally equivalent to the (join/meet/complement) $\mathcal{OL}$ basis {AJ, B1, DM, CC, ONE}.

**Proof OL-SS**

| | | |
|---|---|---|
| 1 | $A \vee (B \vee C) \neq B \vee (A \vee C) \mid \$Ans(AJ)$ | [] |
| 2 | $A \vee c(c(A) \vee c(B)) \neq A \mid \$Ans(B1_rewritten)$ | [] |
| 3 | $A \wedge B \neq c(c(A) \vee c(B)) \mid \$Ans(DM)$ | [] |
| 4 | $c(c(A)) \neq A \mid \$Ans(CC)$ | [] |
| 5 | $A \vee c(A) \neq B \vee c(B) \mid \$Ans(ONE)$ | [] |
| 6 | $(A\|B) \neq c(A) \vee c(B) \mid \$Ans(DEF_SS)$ | [] |
| 8 | $(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$ | [] |
| 9 | $((x\|x)\|(x\|y)) = x$ | [] |
| 11 | $(x\|(x\|x)) = (y\|(y\|y))$ | [] |
| 12 | $x \vee y = ((x\|x)\|(y\|y))$ | [] |
| 13 | $((x\|x)\|(y\|y)) = x \vee y$ | [12] |
| 15 | $x \wedge y = ((x\|y)\|(x\|y))$ | [] |
| 16 | $((x\|y)\|(x\|y)) = x \wedge y$ | [15] |
| 18 | $c(x) = (x\|x)$ | [] |
| 20,19 | $(x\|x) = c(x)$ | [18] |
| 21 | $(x\|c((y\|z))) = (y\|c((x\|z)))$ | [8,20,20] |
| 22 | $(x\|c(x)) = (y\|c(y))$ | [11,20,20] |
| 24,23 | $c((x\|y)) = x \wedge y$ | [16:20] |
| 25 | $(c(x)\|c(y)) = x \vee y$ | [13:20,20] |
| 27 | $(c(x)\|(x\|y)) = x$ | [9:20] |
| 29 | $(x\|y \wedge z) = (y\|x \wedge z)$ | [21,24,24] |

| | | |
|---|---|---|
| 37,36 | $c(c(x)) = x$ | [19 → 27,20] |
| 38 | $\$Ans(CC)$ | [36,4] |
| 41 | $c(x) \wedge (x\|y) = c(x)$ | [27 → 23] |
| 47 | $x \wedge c(x) = y \wedge c(y)$ | [22 → 23,24] |
| 49,48 | $(x\|y) = c(x \wedge y)$ | [23 → 36] |
| 54 | $c(x) \wedge c(x \wedge y) = c(x)$ | [41:49] |
| 58 | $c(x \wedge (y \wedge z)) = c(y \wedge (x \wedge z))$ | [29:49,49] |
| 59 | $c(c(x) \wedge c(y)) = x \vee y$ | [25:49] |
| 64 | $c(x) \wedge x = y \wedge c(y)$ | [36 → 47] |
| 65 | $x \wedge c(x) = c(y) \wedge y$ | [64] |
| 87 | $x \wedge c(c(x) \wedge y) = x$ | [36 → 54,37] |
| 92,91 | $x \wedge c(y \wedge c(y)) = x$ | [64 → 54,37,37] |
| 103 | $c(x \wedge y) = c(y \wedge x)$ | [91 → 58,92] |
| 162 | $x \wedge c(y \wedge c(x)) = x$ | [103 → 87] |
| 182 | $x \wedge y = y \wedge x$ | [103 → 36,37] |
| 205,204 | $c(x \wedge c(y)) = c(x) \vee y$ | [36 → 59] |
| 210,209 | $c(c(x) \wedge y) = x \vee c(y)$ | [36 → 59] |
| 214 | $x \vee c(x) = y \vee c(y)$ | [65 → 59,210] |
| 215 | $\$Ans(ONE)$ | [214,5] |
| 218 | $x \vee y = y \vee x$ | [103 → 59,205,37] |
| 223 | $x \wedge (c(y) \vee x) = x$ | [162:205] |
| 330 | $x \wedge (y \vee x) = x$ | [36 → 223] |
| 340 | $x \wedge (x \vee y) = x$ | [218 → 330] |
| 344 | $(x \vee y) \wedge y = y$ | [182 → 330] |
| 380 | $(x \vee y) \wedge x = x$ | [182 → 340] |
| 386 | $c(x \wedge (y \wedge (x \vee z))) = c(y \wedge x)$ | [340 → 58] |
| 400 | $c((x \vee y) \wedge (z \wedge x)) = c(z \wedge x)$ | [380 → 58] |
| 913,912 | $c(x \wedge y) = c(x) \vee c(y)$ | [36 → 204] |
| 914 | $c(c(x) \vee y) \vee x = x$ | [380 → 204,37] |
| 916 | $c(x \vee c(y)) \vee y = y$ | [344 → 204,37] |
| 1260 | $c(x \vee y) \vee (c(z) \vee c(x)) = c(z) \vee c(x)$ | [400:913,913,913] |
| 1268 | $c(x) \vee (c(y) \vee c(x \vee z)) = c(y) \vee c(x)$ | [386:913,913,913] |
| 1360 | $(x\|y) = c(x) \vee c(y)$ | [48:913] |
| 1362 | $\$Ans(DEF_SS)$ | [1360,6] |
| 1365 | $x \vee c(c(x) \vee y) = x$ | [218 → 914] |
| 1367 | $\$Ans(B1_rewritten)$ | [1365,2] |
| 1372 | $c(x \vee y) \vee c(y) = c(y)$ | [36 → 916] |
| 1374 | $x \vee c(y \vee c(x)) = x$ | [218 → 916] |
| 1461,1460 | $c(x) \vee c(x \vee y) = c(x)$ | [36 → 1365] |
| 1465,1464 | $c(x) \vee c(y \vee x) = c(x)$ | [36 → 1374] |
| 1810 | $x \wedge y = c(c(x) \vee c(y))$ | [912 → 36] |
| 1812 | $\$Ans(DM)$ | [1810,3] |
| 2611 | $c((x \vee y) \vee z) \vee c(y) = c(y)$ | [1464 → 1260,1465] |
| 2613 | $c((x \vee y) \vee z) \vee c(x) = c(x)$ | [1460 → 1260,1461] |
| 2631 | $c(x) \vee (y \vee c(x \vee z)) = y \vee c(x)$ | [36 → 1268,37] |
| 2669 | $((x \vee y) \vee z) \vee y = (x \vee y) \vee z$ | [2611 → 1365,37] |
| 3002,3001 | $((x \vee y) \vee z) \vee x = (x \vee y) \vee z$ | [2613 → 1365,37] |
| 5126 | $(x \vee y) \vee z = (y \vee x) \vee z$ | [218 → 2669,3002] |
| 5169 | $x \vee (y \vee z) = (z \vee y) \vee x$ | [218 → 5126] |
| 14574 | $(x \vee y) \vee (z \vee y) = z \vee (x \vee y)$ | [1372 → 2631,37,37,37] |
| 15295,15294 | $(x \vee y) \vee (z \vee x) = y \vee (z \vee x)$ | [5169 → 14574] |
| 15296 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [5126 → 14574,15295] |
| 15297 | $\$Ans(AJ)$ | [15296,1] |

**Proof OL-SS-2**

| | | |
|---|---|---|
| 1 | $(A\|((B\|C)\|(B\|C))) \neq (B\|((A\|C)\|(A\|C)))$ \| $\$Ans(SS)$ | [] |
| 2 | $((A\|A)\|(A\|B)) \neq A$ \| $\$Ans(SS)$ | [] |
| 3 | $(A\|(A\|A)) \neq (B\|(B\|B))$ \| $\$Ans(SS)$ | [] |
| 4 | $A \vee B \neq ((A\|A)\|(B\|B))$ \| $\$Ans(J)$ | [] |
| 5 | $A \wedge B \neq ((A\|B)\|(A\|B))$ \| $\$Ans(M)$ | [] |
| 6 | $c(A) \neq (A\|A)$ \| $\$Ans(C)$ | [] |
| 8 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [] |
| 9 | $x \vee (x \wedge y) = x$ | [] |
| 12,11 | $x \wedge y = c(c(x) \vee c(y))$ | [] |
| 14,13 | $c(c(x)) = x$ | [] |
| 15 | $x \vee c(x) = y \vee c(y)$ | [] |
| 16 | $(x\|y) = c(x) \vee c(y)$ | [] |
| 18,17 | $c(x) \vee c(y) = (x\|y)$ | [16] |
| 19 | $x \vee c((x\|y)) = x$ | [9:12,18] |
| 21 | $x \wedge y = c((x\|y))$ | [11:18] |
| 27 | $c(x) \vee x = y \vee c(y)$ | [13 → 15] |
| 28 | $x \vee c(x) = c(y) \vee y$ | [27] |
| 31 | $x \vee c(y) = (c(x)\|y)$ | [13 → 17] |
| 39 | $x \vee (y\|z) = c(y) \vee (x \vee c(z))$ | [17 → 8] |
| 40 | $c(x) \vee (y \vee c(z)) = y \vee (x\|z)$ | [39] |
| 48 | $(x\|c(x)) = c(y) \vee y$ | [17 → 28] |
| 49 | $c(x) \vee x = (y\|c(y))$ | [48] |
| 75 | $x \vee y = (c(x)\|c(y))$ | [13 → 31] |
| 77 | $(c(x)\|(x\|y)) = x$ | [19 → 31] |
| 79 | $(c(x)\|c(y)) = x \vee y$ | [75] |
| 86,85 | $c(x) = (x\|x)$ | [77 → 77,14] |
| 87 | $\$Ans(C)$ | [85,6] |
| 94,93 | $x \vee y = ((x\|x)\|(y\|y))$ | [79:86,86] |
| 95 | $\$Ans(J)$ | [93,4] |
| 97,96 | $((x\|x)\|(x\|y)) = x$ | [77:86] |
| 98 | $\$Ans(SS)$ | [96,2] |
| 100 | $(x\|(x\|x)) = (y\|(y\|y))$ | [49:86,94,97,86] |
| 101 | $\$Ans(SS)$ | [100,3] |
| 103 | $(x\|(((y\|y)\|z)\|((y\|y)\|z))) = ((y\|y)\|((x\|z)\|(x\|z)))$ | [40:86,86,94,97,94,97,94] |
| 108 | $x \wedge y = ((x\|y)\|(x\|y))$ | [21:86] |
| 110 | $\$Ans(M)$ | [108,5] |
| 119 | $(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$ | [96 → 103,97,97] |
| 120 | $\$Ans(SS)$ | [119,1] |

The following three Mace2 jobs show that the $\mathcal{OL}$ 3-basis {A_SS, B_SS, ONE_SS} is independent.

**Countermodel OL-SS-a**

```
f(f(x,x),f(x,y)) = x.                              % B_SS
f(x,f(x,x)) = f(y,f(y,y)).                         % ONE_SS
f(A,f(f(B,C),f(B,C))) != f(B,f(f(A,C),f(A,C))).  % denial of A_SS
```

C: 0 A: 0 B: 1

| f: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 2 | 2 | 0 |
| 2 | 1 | 0 | 1 |

Table 18: OL-SS-a.out

**Countermodel OL-SS-b**

```
f(x,f(f(y,z),f(y,z))) = f(y,f(f(x,z),f(x,z))).   % A_SS
f(x,f(x,x)) = f(y,f(y,y)).                       % ONE_SS
f(f(A,A),f(A,B)) != A.                           % denial of B_SS
```

B: 0 A: 1

| f: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |

Table 19: OL-SS-b.out

**Countermodel OL-SS-c**

```
f(x,f(f(y,z),f(y,z))) = f(y,f(f(x,z),f(x,z))).   % A_SS
f(f(x,x),f(x,y)) = x.                            % B_SS
f(A,f(A,A)) != f(B,f(B,B)).                      % denial of ONE_SS
```

B: 0 A: 1

| f: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 1 | 0 | 2 | 1 |
| 2 | 1 | 1 | 1 |

Table 20: OL-SS-c.out

### 2.2.2 Orthomodular Lattices in Terms of the Sheffer Stroke

The following is a 3-basis for orthomodular lattices in terms of the Sheffer stroke.

$(x|((y|z)|(y|z))) = (y|((x|z)|(x|z)))$ % A_SS
$((x|x)|(x|y)) = x$ % B_SS
$(x|(x|(x|y))) = (x|y)$ % OM_SS

The following two Otter jobs show that this basis is definitionally equivalent to the (join/meet/complement) $\mathcal{OML}$ basis {AJ, B1, DM, OM}.

**Proof OML-SS**

| | | |
|---|---|---|
| 1 | $A \vee (B \vee C) \neq B \vee (A \vee C) \mid \$Ans(AJ)$ | [] |
| 2 | $A \vee (A \wedge B) \neq A \mid \$Ans(B1)$ | [] |
| 3 | $A \wedge B \neq c(c(A) \vee c(B)) \mid \$Ans(DM)$ | [] |
| 4 | $A \vee c(A \vee c(A \vee B)) \neq A \vee B \mid \$Ans(OM_rewritten)$ | [] |
| 5 | $(A\|B) \neq c(A) \vee c(B) \mid \$Ans(DEF_SS)$ | [] |

| | | |
|---|---|---|
| 7 | $(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$ | [] |
| 8 | $((x\|x)\|(x\|y)) = x$ | [] |
| 10 | $(x\|(x\|(x\|y))) = (x\|y)$ | [] |
| 12 | $x \vee y = ((x\|x)\|(y\|y))$ | [] |
| 13 | $((x\|x)\|(y\|y)) = x \vee y$ | [12] |
| 15 | $x \wedge y = ((x\|y)\|(x\|y))$ | [] |
| 16 | $((x\|y)\|(x\|y)) = x \wedge y$ | [15] |
| 18 | $c(x) = (x\|x)$ | [] |
| 20,19 | $(x\|x) = c(x)$ | [18] |
| 21 | $(x\|c((y\|z))) = (y\|c((x\|z)))$ | [7,20,20] |
| 23,22 | $c((x\|y)) = x \wedge y$ | [16:20] |
| 25,24 | $(c(x)\|c(y)) = x \vee y$ | [13:20,20] |
| 26 | $(c(x)\|(x\|y)) = x$ | [8:20] |
| 28 | $(x\|y \wedge z) = (y\|x \wedge z)$ | [21,23,23] |
| 39,38 | $c(c(x)) = x \vee x$ | [19 → 24] |
| 44,43 | $c(x) \wedge c(y) = c(x \vee y)$ | [24 → 22] |
| 45 | $(c(x)\|(c(x)\|x \vee y)) = x \vee y$ | [24 → 10,25] |
| 49 | $(x\|y) \vee (x\|y) = c(x \wedge y)$ | [22 → 38] |
| 64,63 | $x \vee x = x$ | [19 → 26,20,39] |
| 74,73 | $(x\|y) = c(x \wedge y)$ | [49:64] |
| 78,77 | $c(c(x)) = x$ | [38:64] |
| 85 | $x \vee (c(x) \wedge (x \vee y)) = x \vee y$ | [45:74,74,44,78] |
| 89 | $x \vee (x \wedge y) = x$ | [26:74,74,44,78] |
| 91 | $\$Ans(B1)$ | [89,2] |
| 100 | $c(x \wedge (y \wedge z)) = c(y \wedge (x \wedge z))$ | [28:74,74] |
| 104,103 | $c(x) \wedge y = c(x \vee c(y))$ | [77 → 43] |
| 107 | $x \vee c(x \vee c(x \vee y)) = x \vee y$ | [85:104] |
| 109 | $\$Ans(OM_rewritten)$ | [107,4] |
| 117,116 | $x \wedge y = c(c(x) \vee c(y))$ | [77 → 103] |
| 118 | $\$Ans(DM)$ | [116,3] |
| 125 | $c(x) \vee (c(y) \vee c(z)) = c(y) \vee (c(x) \vee c(z))$ | [100:117,117,78,78,117,117,78,78] |
| 128 | $(x\|y) = c(x) \vee c(y)$ | [73:117,78] |
| 130 | $\$Ans(DEF_SS)$ | [128,5] |
| 143 | $x \vee (c(y) \vee c(z)) = c(y) \vee (x \vee c(z))$ | [77 → 125,78] |
| 172 | $x \vee (y \vee c(z)) = y \vee (x \vee c(z))$ | [77 → 143,78] |
| 209 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [77 → 172,78] |
| 210 | $\$Ans(AJ)$ | [209,1] |

**Proof OML-SS-2**

| | | |
|---|---|---|
| 1 | $(A\|((B\|C)\|(B\|C))) \neq (B\|((A\|C)\|(A\|C))) \mid \$Ans(A_SS)$ | [] |
| 2 | $((A\|A)\|(A\|B)) \neq A \mid \$Ans(B_SS)$ | [] |
| 3 | $(A\|(A\|(A\|B))) \neq (A\|B) \mid \$Ans(OM_SS)$ | [] |
| 4 | $A \vee B \neq ((A\|A)\|(B\|B)) \mid \$Ans(DEF_J)$ | [] |
| 5 | $A \wedge B \neq ((A\|B)\|(A\|B)) \mid \$Ans(DEF_M)$ | [] |
| 6 | $c(A) \neq (A\|A) \mid \$Ans(DEF_C)$ | [] |
| 8 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [] |
| 9 | $x \vee (x \wedge y) = x$ | [] |
| 12,11 | $x \wedge y = c(c(x) \vee c(y))$ | [] |
| 13 | $x \vee (c(x) \wedge (x \vee y)) = x \vee y$ | [] |
| 14 | $x \vee c(c(c(x)) \vee c(x \vee y)) = x \vee y$ | [13,12] |
| 16 | $(x\|y) = c(x) \vee c(y)$ | [] |

| | | |
|---|---|---|
| 18,17 | $c(x) \vee c(y) = (x\|y)$ | [16] |
| 20,19 | $x \vee c((x\|y)) = x$ | [9:12,18] |
| 21 | $x \vee c((c(x)\|x \vee y)) = x \vee y$ | [14:18] |
| 23 | $x \wedge y = c((x\|y))$ | [11:18] |
| 25 | $x \vee (y \vee (z \vee u)) = z \vee (x \vee (y \vee u))$ | [8 → 8] |
| 27 | $x \vee (y \vee c((x\|z))) = y \vee x$ | [19 → 8] |
| 29 | $(x\|(c(x)\|y)) = c(x)$ | [19 → 17] |
| 31 | $x \vee (y\|z) = c(y) \vee (x \vee c(z))$ | [17 → 8] |
| 32 | $c(x) \vee (y \vee c(z)) = y \vee (x\|z)$ | [31] |
| 35 | $x \vee c(c(x)) = x$ | [29 → 19] |
| 37 | $x \vee c((c(x)\|x)) = x$ | [19 → 21,20] |
| 39 | $(x\|(c(c(x))\|(x\|y))) = (x\|y)$ | [17 → 21,18,18] |
| 47 | $x \vee (y \vee c(c(x))) = y \vee x$ | [35 → 8] |
| 53 | $x \vee (y\|c(x)) = c(y) \vee x$ | [17 → 47] |
| 54 | $c(x) \vee y = y \vee (x\|c(y))$ | [53] |
| 92,91 | $c(x) \vee (y\|c(c(x))) = (y\|x)$ | [17 → 54] |
| 104 | $c(x) \vee x = x \vee c(x)$ | [37 → 27] |
| 107,106 | $c(x) \vee (x \vee y) = x \vee c(x)$ | [21 → 27] |
| 140 | $x \vee (y \vee c(y)) = c(y) \vee (x \vee y)$ | [104 → 8] |
| 144 | $c(x) \vee (y \vee x) = y \vee (x \vee c(x))$ | [140] |
| 149 | $c(c(x)) \vee (x \vee c(x)) = (x\|c(x))$ | [106 → 106,18] |
| 156,155 | $c(x) \vee (y \vee x) = x \vee c(x)$ | [47 → 106] |
| 170,169 | $x \vee (y \vee c(y)) = y \vee c(y)$ | [25 → 106,107] |
| 171 | $x \vee (c(x) \vee y) = x \vee c(x)$ | [8 → 106] |
| 174,173 | $x \vee c(x) = (x\|c(x))$ | [149:170] |
| 176,175 | $x \vee (y\|c(y)) = (y\|c(y))$ | [144:156,174,174] |
| 183 | $x \vee (c(x) \vee y) = (x\|c(x))$ | [171:174] |
| 237 | $(x\|c(x)) = (y\|c(y))$ | [175 → 183,176] |
| 252 | $c(x) \vee (y\|z) = c(y) \vee (x\|z)$ | [54 → 32,92] |
| 270 | $x \vee (y \vee c((z\|c(z)))) = y \vee x$ | [237 → 27] |
| 272,271 | $x \vee c((y\|c(y))) = x$ | [237 → 19] |
| 274 | $x \vee y = y \vee x$ | [270,272] |
| 279 | $x \vee (y\|c(x)) = x \vee c(y)$ | [54 → 274] |
| 280 | $c(x) \vee y = (x\|c(y)) \vee y$ | [53 → 274] |
| 283 | $c(c(x)) \vee x = x$ | [35 → 274] |
| 294 | $c((x\|y)) \vee x = x$ | [19 → 274] |
| 296 | $(x\|y) = (y\|x)$ | [17 → 274,18] |
| 306 | $x \vee (c((x\|y)) \vee z) = z \vee x$ | [274 → 27] |
| 314 | $x \vee y = y \vee (c((y\|z)) \vee x)$ | [306] |
| 330 | $(c(c(x))\|x) = c(x)$ | [17 → 283] |
| 352,351 | $((c(x)\|y)\|x) = c(x)$ | [29 → 296] |
| 353 | $x \vee (c(x)\|y) = c(y) \vee x$ | [296 → 53] |
| 354 | $x \vee (y\|z) = c(z) \vee (x \vee c(y))$ | [296 → 31] |
| 363 | $c(x) \vee y = y \vee (c(y)\|x)$ | [353] |
| 375 | $c((x\|y)) \vee y = y$ | [296 → 294] |
| 378,377 | $c((x\|c(x))) \vee y = y$ | [237 → 294] |
| 379 | $x \vee ((x\|y)\|c(x)) = x$ | [54 → 294] |
| 409,408 | $c(c(x)) = (c(x)\|c(x))$ | [330 → 29] |
| 472 | $(x\|((c(x)\|c(x))\|(x\|y))) = (x\|y)$ | [39:409] |
| 480 | $((x\|c(x))\|y) = c(y)$ | [237 → 375,18] |
| 493 | $((x\|c(y))\|y) = c(y)$ | [17 → 375] |
| 497 | $c(x) = ((y\|c(y))\|x)$ | [480] |
| 643 | $c(((c((x\|c(x)))\|c((x\|c(x))))\|y)) = y$ | [21 → 377,378,409,378] |
| 766,765 | $c(((x\|c(x))\|y)) = (c(y)\|c(y))$ | [497 → 408] |

| | | |
|---|---|---|
| 898 | $x \vee (c(x)\|(x\|y)) = x$ | [296 → 379] |
| 900 | $((x\|y)\|c(x)) \vee x = x$ | [274 → 379] |
| 1132 | $(c(x)\|(x\|y)) \vee x = x$ | [274 → 898] |
| 1162,1161 | $((c(x)\|y)\|(c(x)\|c(x))) \vee (z\|x) = (z\|x)$ | [900 → 32,18,409] |
| 2136 | $(c((x\|c(y)))\|c(y)) \vee (x\|c(y)) = (x\|c(y))$ | [493 → 1132] |
| 2138 | $(c((c(x)\|y))\|c(x)) \vee (c(x)\|y) = (c(x)\|y)$ | [351 → 1132] |
| 3290 | $x \vee (c(x)\|y) = x \vee c(y)$ | [296 → 279] |
| 4012 | $(c(((c(x)\|y)\|(c(x)\|c(x))))\|x) = c(x)$ | [379 → 363,409,409,1162] |
| 4016 | $(x\|c(y)) \vee y = y \vee (c(y)\|x)$ | [280 → 363] |
| 5787,5786 | $((c((x\|c(x)))\|c((x\|c(x))))\|y) = ((x\|c(x))\|y)$ | [377 → 3290,409,18] |
| 5788 | $(c(x)\|y) \vee x = x \vee c(y)$ | [274 → 3290] |
| 5806,5805 | $(c(x)\|c(x)) = x$ | [643:5787,766] |
| 5807 | $x \vee c(y) = (c(x)\|y) \vee x$ | [5788] |
| 5841,5840 | $c(x) = (x\|x)$ | [4012:5806,352,409,5806] |
| 5842 | $\$Ans(DEF_C)$ | [5840,6] |
| 5938 | $(x\|(((x\|x)\|(x\|x))\|(x\|y))) = (x\|y)$ | [472:5841,5841] |
| 5963,5962 | $((x\|x)\|y) \vee x = x \vee (y\|y)$ | [5807,5841,5841] |
| 5995,5994 | $((x\|x)\|(x\|x)) = x$ | [5805:5841,5841] |
| 6607,6606 | $(x\|(y\|y)) \vee y = y \vee ((y\|y)\|x)$ | [4016:5841,5841] |
| 7277,7276 | $x \vee (y\|y) = ((x\|x)\|y)$ | [2138:5841,5841,5841,5841,5963,5995,5963,5841] |
| 7279,7278 | $x \vee ((x\|x)\|y) = (y\|(x\|x))$ | [2136:5841,5841,5841,5841,5963,5995,6607,5841] |
| 7708,7707 | $((x\|x)\|(x\|y)) = x$ | [1132:5841,5963,7277] |
| 7709 | $\$Ans(B_SS)$ | [7707,2] |
| 7950,7949 | $(x\|x) \vee y = (x\|(y\|y))$ | [363:5841,5841,7279] |
| 7957,7956 | $x \vee (y\|z) = (z\|(((x\|x)\|y)\|((x\|x)\|y)))$ | [354:5841,5841,7277,7950] |
| 7961,7960 | $x \vee y = ((x\|x)\|(y\|y))$ | [314:5841,7950,7957,7708,7708] |
| 7962 | $\$Ans(DEF_J)$ | [7960,4] |
| 7967 | $(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$ | [252:5841,7961,5995,5841,7961,5995] |
| 7968 | $\$Ans(A_SS)$ | [7967,1] |
| 7977 | $x \wedge y = ((x\|y)\|(x\|y))$ | [23:5841] |
| 7979 | $\$Ans(DEF_M)$ | [7977,5] |
| 8024 | $(x\|(x\|(x\|y))) = (x\|y)$ | [5938:5995] |
| 8026 | $\$Ans(OM_SS)$ | [8024,3] |

The following three Mace2 jobs show that the $\mathcal{OML}$ 3-basis {A_SS, B_SS, OM_SS} is independent.

**Countermodel OML-SS-a**

```
f(f(x,x),f(x,y)) = x.                              % B_SS
f(x,f(x,f(x,y))) = f(x,y).                         % OM_SS
f(A,f(f(B,C),f(B,C))) != f(B,f(f(A,C),f(A,C))).  % denial of A_SS
```

C: 0 A: 0 B: 1

| f: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |

Table 21: OML-SS-a.out

**Countermodel OML-SS-b**

```
f(x,f(f(y,z),f(y,z))) = f(y,f(f(x,z),f(x,z))).   % A_SS
```

```
f(x,f(x,f(x,y))) = f(x,y).                          % OM_SS
f(f(A,A),f(A,B)) != A.                              % denial of B_SS
```

B: 0 A: 1

| f: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |

Table 22: OML-SS-b.out

**Countermodel OML-SS-c**

```
f(x,f(f(y,z),f(y,z))) = f(y,f(f(x,z),f(x,z))).   % A_SS
f(f(x,x),f(x,y)) = x.                             % B_SS
f(A,f(A,f(A,B))) != f(A,B).                       % denial of OM_SS
```

B: 0 A: 1

| f: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 2 | 2 | 2 |
| 1 | 2 | 1 | 1 |
| 2 | 2 | 1 | 0 |

Table 23: OML-SS-c.out

### 2.2.3 Modular Ortholattices in Terms of the Sheffer Stroke

The following is a 4-basis for modular ortholattices in terms of the Sheffer stroke.

$(x|((y|z)|(y|z))) = (y|((x|z)|(x|z)))$ % A_SS
$((x|x)|(x|y)) = x$ % B_SS
$(x|(x|x)) = (y|(y|y))$ % ONE_SS
$(x|(y|(x|(z|z)))) = (x|(z|(x|(y|y))))$ % MOD_SS

The following two Otter jobs show that this basis is definitionally equivalent to the (join/meet/complement) $\mathcal{MOL}$ basis {AJ, B1, DM, CC, ONE, MOD}.

**Proof MOL-SS**

| | | |
|---|---|---|
| 1 | $A \vee (B \vee C) \neq B \vee (A \vee C) \mid \$Ans(AJ)$ | [] |
| 2 | $A \vee (A \wedge B) \neq A \mid \$Ans(B1)$ | [] |
| 3 | $A \wedge B \neq c(c(A) \vee c(B)) \mid \$Ans(DM)$ | [] |
| 4 | $c(c(A)) \neq A \mid \$Ans(CC)$ | [] |
| 5 | $A \vee c(A) \neq B \vee c(B) \mid \$Ans(ONE)$ | [] |
| 6 | $A \vee c(c(B) \vee c(A \vee C)) \neq A \vee c(c(C) \vee c(A \vee B)) \mid \$Ans(MOD_rewritten)$ | [] |
| 7 | $(A\|B) \neq c(A) \vee c(B) \mid \$Ans(DEF_SS)$ | [] |
| 9 | $(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$ | [] |
| 10 | $((x\|x)\|(x\|y)) = x$ | [] |
| 12 | $(x\|(x\|x)) = (y\|(y\|y))$ | [] |
| 13 | $(x\|(y\|(x\|(z\|z)))) = (x\|(z\|(x\|(y\|y))))$ | [] |
| 14 | $x \vee y = ((x\|x)\|(y\|y))$ | [] |
| 15 | $((x\|x)\|(y\|y)) = x \vee y$ | [14] |
| 17 | $x \wedge y = ((x\|y)\|(x\|y))$ | [] |
| 18 | $((x\|y)\|(x\|y)) = x \wedge y$ | [17] |

| | | |
|---|---|---|
| 20 | $c(x) = (x\|x)$ | [] |
| 22,21 | $(x\|x) = c(x)$ | [20] |
| 23 | $(x\|c((y\|z))) = (y\|c((x\|z)))$ | [9,22,22] |
| 24 | $(x\|c(x)) = (y\|c(y))$ | [12,22,22] |
| 25 | $(x\|(y\|(x\|c(z)))) = (x\|(z\|(x\|c(y))))$ | [13,22,22] |
| 27,26 | $c((x\|y)) = x \wedge y$ | [18:22] |
| 28 | $(c(x)\|c(y)) = x \vee y$ | [15:22,22] |
| 30 | $(c(x)\|(x\|y)) = x$ | [10:22] |
| 32 | $(x\|y \wedge z) = (y\|x \wedge z)$ | [23,27,27] |
| 40,39 | $c(c(x)) = x$ | [21 → 30,22] |
| 41 | $\$Ans(CC)$ | [39,4] |
| 44 | $c(x) \wedge (x\|y) = c(x)$ | [30 → 26] |
| 50 | $x \wedge c(x) = y \wedge c(y)$ | [24 → 26,27] |
| 52,51 | $(x\|y) = c(x \wedge y)$ | [26 → 39] |
| 57 | $c(x) \wedge c(x \wedge y) = c(x)$ | [44:52] |
| 61 | $c(x \wedge (y \wedge z)) = c(y \wedge (x \wedge z))$ | [32:52,52] |
| 62 | $c(c(x) \wedge c(y)) = x \vee y$ | [28:52] |
| 64 | $c(x \wedge c(y \wedge c(x \wedge c(z)))) = c(x \wedge c(z \wedge c(x \wedge c(y))))$ | [25:52,52,52,52,52,52] |
| 68 | $c(x) \wedge x = y \wedge c(y)$ | [39 → 50] |
| 69 | $x \wedge c(x) = c(y) \wedge y$ | [68] |
| 91 | $x \wedge c(c(x) \wedge y) = x$ | [39 → 57,40] |
| 96,95 | $x \wedge c(y \wedge c(y)) = x$ | [68 → 57,40,40] |
| 107 | $c(x \wedge y) = c(y \wedge x)$ | [95 → 61,96] |
| 166 | $x \wedge c(y \wedge c(x)) = x$ | [107 → 91] |
| 186 | $x \wedge y = y \wedge x$ | [107 → 39,40] |
| 209,208 | $c(x \wedge c(y)) = c(x) \vee y$ | [39 → 62] |
| 214,213 | $c(c(x) \wedge y) = x \vee c(y)$ | [39 → 62] |
| 215 | $x \vee (x \wedge y) = x$ | [91 → 62,40,40] |
| 217 | $\$Ans(B1)$ | [215,2] |
| 219 | $x \vee c(x) = y \vee c(y)$ | [69 → 62,214] |
| 220 | $\$Ans(ONE)$ | [219,5] |
| 223 | $x \vee y = y \vee x$ | [107 → 62,209,40] |
| 228 | $x \wedge (c(y) \vee x) = x$ | [166:209] |
| 240 | $c(x) \vee (y \wedge (c(x) \vee z)) = c(x) \vee (z \wedge (c(x) \vee y))$ | [64:209,209,209,209] |
| 336 | $x \wedge (y \vee x) = x$ | [39 → 228] |
| 346 | $x \wedge (x \vee y) = x$ | [223 → 336] |
| 350 | $(x \vee y) \wedge y = y$ | [186 → 336] |
| 386 | $(x \vee y) \wedge x = x$ | [186 → 346] |
| 392 | $c(x \wedge (y \wedge (x \vee z))) = c(y \wedge x)$ | [346 → 61] |
| 406 | $c((x \vee y) \wedge (z \wedge x)) = c(z \wedge x)$ | [386 → 61] |
| 919,918 | $c(x \wedge y) = c(x) \vee c(y)$ | [39 → 208] |
| 920 | $c(c(x) \vee y) \vee x = x$ | [386 → 208,40] |
| 922 | $c(x \vee c(y)) \vee y = y$ | [350 → 208,40] |
| 1266 | $c(x \vee y) \vee (c(z) \vee c(x)) = c(z) \vee c(x)$ | [406:919,919,919] |
| 1274 | $c(x) \vee (c(y) \vee c(x \vee z)) = c(y) \vee c(x)$ | [392:919,919,919] |
| 1366 | $(x\|y) = c(x) \vee c(y)$ | [51:919] |
| 1368 | $\$Ans(DEF_SS)$ | [1366,7] |
| 1371 | $x \vee c(c(x) \vee y) = x$ | [223 → 920] |
| 1377 | $c(x \vee y) \vee c(y) = c(y)$ | [39 → 922] |
| 1379 | $x \vee c(y \vee c(x)) = x$ | [223 → 922] |
| 1466,1465 | $c(x) \vee c(x \vee y) = c(x)$ | [39 → 1371] |
| 1470,1469 | $c(x) \vee c(y \vee x) = c(x)$ | [39 → 1379] |
| 1816,1815 | $x \wedge y = c(c(x) \vee c(y))$ | [918 → 39] |
| 1817 | $\$Ans(DM)$ | [1815,3] |

| | | |
|---|---|---|
| 2100 | $c(x) \vee c(c(y) \vee c(c(x) \vee z)) = c(x) \vee c(c(z) \vee c(c(x) \vee y))$ | [240:1816,1816] |
| 2617 | $c((x \vee y) \vee z) \vee c(y) = c(y)$ | [1469 → 1266,1470] |
| 2619 | $c((x \vee y) \vee z) \vee c(x) = c(x)$ | [1465 → 1266,1466] |
| 2637 | $c(x) \vee (y \vee c(x \vee z)) = y \vee c(x)$ | [39 → 1274,40] |
| 2675 | $((x \vee y) \vee z) \vee y = (x \vee y) \vee z$ | [2617 → 1371,40] |
| 3006,3005 | $((x \vee y) \vee z) \vee x = (x \vee y) \vee z$ | [2619 → 1371,40] |
| 5132 | $(x \vee y) \vee z = (y \vee x) \vee z$ | [223 → 2675,3006] |
| 5175 | $x \vee (y \vee z) = (z \vee y) \vee x$ | [223 → 5132] |
| 7211 | $x \vee c(c(y) \vee c(x \vee z)) = x \vee c(c(z) \vee c(x \vee y))$ | [39 → 2100,40,40,40] |
| 7212 | $\$Ans(MOD_rewritten)$ | [7211,6] |
| 14692 | $(x \vee y) \vee (z \vee y) = z \vee (x \vee y)$ | [1377 → 2637,40,40,40] |
| 15413,15412 | $(x \vee y) \vee (z \vee x) = y \vee (z \vee x)$ | [5175 → 14692] |
| 15414 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [5132 → 14692,15413] |
| 15415 | $\$Ans(AJ)$ | [15414,1] |

**Proof MOL-SS-2**

| | | |
|---|---|---|
| 1 | $(A\|((B\|C)\|(B\|C))) \neq (B\|((A\|C)\|(A\|C))) \mid \$Ans(A_SS)$ | [] |
| 2 | $((A\|A)\|(A\|B)) \neq A \mid \$Ans(B_SS)$ | [] |
| 3 | $(A\|(A\|A)) \neq (B\|(B\|B)) \mid \$Ans(ONE_SS)$ | [] |
| 4 | $(A\|(B\|(A\|(C\|C)))) \neq (A\|(C\|(A\|(B\|B)))) \mid \$Ans(MOD_SS)$ | [] |
| 5 | $A \vee B \neq ((A\|A)\|(B\|B)) \mid \$Ans(DEF_J)$ | [] |
| 6 | $A \wedge B \neq ((A\|B)\|(A\|B)) \mid \$Ans(DEF_M)$ | [] |
| 7 | $c(A) \neq (A\|A) \mid \$Ans(DEF_C)$ | [] |
| 9 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [] |
| 10 | $x \vee (x \wedge y) = x$ | [] |
| 13,12 | $x \wedge y = c(c(x) \vee c(y))$ | [] |
| 15,14 | $c(c(x)) = x$ | [] |
| 16 | $x \vee c(x) = y \vee c(y)$ | [] |
| 17 | $x \vee (y \wedge (x \vee z)) = x \vee (z \wedge (x \vee y))$ | [] |
| 18 | $x \vee c(c(y) \vee c(x \vee z)) = x \vee c(c(z) \vee c(x \vee y))$ | [17,13,13] |
| 19 | $(x\|y) = c(x) \vee c(y)$ | [] |
| 21,20 | $c(x) \vee c(y) = (x\|y)$ | [19] |
| 22 | $x \vee c((x\|y)) = x$ | [10:13,21] |
| 24 | $x \vee c((y\|x \vee z)) = x \vee c((z\|x \vee y))$ | [18,21,21] |
| 25 | $x \wedge y = c((x\|y))$ | [12:21] |
| 31 | $c(x) \vee x = y \vee c(y)$ | [14 → 16] |
| 32 | $x \vee c(x) = c(y) \vee y$ | [31] |
| 35 | $x \vee c(y) = (c(x)\|y)$ | [14 → 20] |
| 37 | $(x\|(c(x)\|y)) = c(x)$ | [22 → 20] |
| 43 | $x \vee (y\|z) = c(y) \vee (x \vee c(z))$ | [20 → 9] |
| 44 | $c(x) \vee (y \vee c(z)) = y \vee (x\|z)$ | [43] |
| 52 | $(x\|c(x)) = c(y) \vee y$ | [20 → 32] |
| 53 | $c(x) \vee x = (y\|c(y))$ | [52] |
| 60 | $(x\|(y\|(x\|z))) = (x\|(c(z)\|c(x) \vee y))$ | [20 → 24,21,21] |
| 65 | $(x\|(y\|c(x) \vee z)) = (x\|(z\|c(x) \vee y))$ | [20 → 24,21] |
| 68 | $(x\|(c(y)\|c(x) \vee z)) = (x\|(z\|(x\|y)))$ | [60] |
| 73 | $x \vee y = (c(x)\|c(y))$ | [14 → 35] |
| 77 | $(c(x)\|(x\|y)) = x$ | [22 → 35] |
| 79 | $(c(x)\|c(y)) = x \vee y$ | [73] |
| 93,92 | $c(x) = (x\|x)$ | [77 → 77,15] |
| 94 | $\$Ans(DEF_C)$ | [92,7] |

| | | |
|---|---|---|
| 100,99 | $x \vee y = ((x\|x)\|(y\|y))$ | [79:93,93] |
| 101 | $\$Ans(DEF_J)$ | [99,5] |
| 103,102 | $((x\|x)\|(x\|y)) = x$ | [77:93] |
| 104 | $\$Ans(B_SS)$ | [102,2] |
| 106 | $(x\|((y\|y)\|(x\|(z\|z)))) = (x\|(z\|(x\|y)))$ | [68:93,93,100,103] |
| 108 | $(x\|(y\|(x\|(z\|z)))) = (x\|(z\|(x\|(y\|y))))$ | [65:93,100,103,93,100,103] |
| 109 | $\$Ans(MOD_SS)$ | [108,4] |
| 112 | $(x\|(x\|x)) = (y\|(y\|y))$ | [53:93,100,103,93] |
| 113 | $\$Ans(ONE_SS)$ | [112,3] |
| 115 | $(x\|(((y\|y)\|z)\|((y\|y)\|z))) = ((y\|y)\|((x\|z)\|(x\|z)))$ | [44:93,93,100,103,100,103,100] |
| 119,118 | $(x\|((x\|x)\|y)) = (x\|x)$ | [37:93,93] |
| 120 | $x \wedge y = ((x\|y)\|(x\|y))$ | [25:93] |
| 122 | $\$Ans(DEF_M)$ | [120,6] |
| 145 | $((x\|x)\|(y\|x)) = x$ | [106 → 108,103,103,103,119,103] |
| 176 | $(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$ | [145 → 115,103,103] |
| 177 | $\$Ans(A_SS)$ | [176,1] |

The following four Mace2 jobs show that $\mathcal{MOL}$ 4-basis {A_SS, B_SS, ONE_SS, MOD_SS} is independent.

**Countermodel MOL-SS-a**

```
f(f(x,x),f(x,y)) = x.                                % B_SS
f(x,f(x,x)) = f(y,f(y,y)).                           % ONE_SS
f(x,f(y,f(x,f(z,z)))) = f(x,f(z,f(x,f(y,y)))).       % MOD_SS
f(A,f(f(B,C),f(B,C))) != f(B,f(f(A,C),f(A,C))).      % denial of A_SS
```

C: 0 A: 0 B: 1

| f: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 2 | 2 | 0 |
| 2 | 1 | 0 | 1 |

Table 24: MOL-SS-a.out

**Countermodel MOL-SS-b**

```
f(x,f(f(y,z),f(y,z))) = f(y,f(f(x,z),f(x,z))).       % A_SS
f(x,f(x,x)) = f(y,f(y,y)).                           % ONE_SS
f(x,f(y,f(x,f(z,z)))) = f(x,f(z,f(x,f(y,y)))).       % MOD_SS
f(f(A,A),f(A,B)) != A.                               % denial of B_SS
```

B: 0 A: 1

| f: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |

Table 25: MOL-SS-b.out

**Countermodel MOL-SS-c**

```
f(x,f(f(y,z),f(y,z))) = f(y,f(f(x,z),f(x,z))).       % A_SS
```

```
f(f(x,x),f(x,y)) = x.                                  % B_SS
f(x,f(y,f(x,f(z,z)))) = f(x,f(z,f(x,f(y,y)))).         % MOD_SS
f(A,f(A,A)) != f(B,f(B,B)).                            % denial of ONE_SS
```

B: 0 A: 1

| f: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 1 | 0 | 2 | 1 |
| 2 | 1 | 1 | 1 |

Table 26: MOL-SS-c.out

**Countermodel MOL-SS-d**

```
f(x,f(f(y,z),f(y,z))) = f(y,f(f(x,z),f(x,z))).   % A_SS
f(f(x,x),f(x,y)) = x.                            % B_SS
f(x,f(x,x)) = f(y,f(y,y)).                       % ONE_SS
f(A,f(B,f(A,f(C,C)))) != f(A,f(C,f(A,f(B,B)))).  % denial of MOD_SS
```

B: 0 A: 0 C: 1

| f: | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 2 | 3 | 3 | 2 | 3 | 1 |
| 1 | 3 | 5 | 0 | 5 | 3 | 3 |
| 2 | 3 | 0 | 0 | 0 | 3 | 3 |
| 3 | 2 | 5 | 0 | 4 | 3 | 1 |
| 4 | 3 | 3 | 3 | 3 | 3 | 3 |
| 5 | 1 | 3 | 3 | 1 | 3 | 1 |

Table 27: MOL-SS-d.out

### 2.2.4 Boolean Algebra in Terms of the Sheffer Stroke

The following is a 2-basis for Boolean algebra in terms of the Sheffer stroke.

$(x|((y|z)|(y|z))) = (y|((x|z)|(x|z)))$ % A_SS
$((x|(y|y))|(x|y)) = x$ % CUT_SS

The two Otter job below show that this basis is definitionally equivalent to the (join/meet/complement) $\mathcal{BA}$ basis {AJ, DM, ONE, CUT}.

For reference, the simplest multiequation basis for $\mathcal{BA}$ in terms of the Sheffer stroke is known to be the following [20].

$x|y = y|x$
$(x|y)|(x|(y|z)) = x$

**Proof BA-SS**

| | | |
|---|---|---|
| 1 | $A \vee (B \vee C) \neq B \vee (A \vee C) \mid \$Ans(AJ)$ | [] |
| 2 | $A \wedge B \neq c(c(A) \vee c(B)) \mid \$Ans(DM)$ | [] |
| 3 | $A \vee c(A) \neq B \vee c(B) \mid \$Ans(ONE)$ | [] |
| 4 | $c(c(A \vee c(B)) \vee c(A \vee B)) \neq A \mid \$Ans(CUT_rewritten)$ | [] |
| 5 | $(A\|B) \neq c(A) \vee c(B) \mid \$Ans(DEF_SS)$ | [] |

| | | |
|---|---|---|
| 7 | $(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$ | [] |
| 8 | $((x\|(y\|y))\|(x\|y)) = x$ | [] |
| 10 | $x \vee y = ((x\|x)\|(y\|y))$ | [] |
| 11 | $((x\|x)\|(y\|y)) = x \vee y$ | [10] |
| 13 | $x \wedge y = ((x\|y)\|(x\|y))$ | [] |
| 14 | $((x\|y)\|(x\|y)) = x \wedge y$ | [13] |
| 16 | $c(x) = (x\|x)$ | [] |
| 18,17 | $(x\|x) = c(x)$ | [16] |
| 19 | $(x\|c((y\|z))) = (y\|c((x\|z)))$ | [7,18,18] |
| 21,20 | $c((x\|y)) = x \wedge y$ | [14:18] |
| 23,22 | $(c(x)\|c(y)) = x \vee y$ | [11:18,18] |
| 24 | $((x\|c(y))\|(x\|y)) = x$ | [8:18] |
| 26 | $(x\|y \wedge z) = (y\|x \wedge z)$ | [19,21,21] |
| 27 | $x \wedge x = c(c(x))$ | [17 → 20] |
| 30,29 | $(x \wedge y\|c(z)) = (x\|y) \vee z$ | [20 → 22] |
| 31 | $(c(x)\|y \wedge z) = x \vee (y\|z)$ | [20 → 22] |
| 33,32 | $c(c(x)) = x \vee x$ | [17 → 22] |
| 36,35 | $x \wedge x = x \vee x$ | [27:33] |
| 38,37 | $c(x) \wedge c(y) = c(x \vee y)$ | [22 → 20] |
| 39 | $((x\|y \wedge z)\|(x\|(y\|z))) = x$ | [20 → 24] |
| 41 | $(x \vee y\|(c(x)\|y)) = c(x)$ | [22 → 24] |
| 43 | $(((x\|c(y))\|x \wedge y)\|x) = (x\|c(y))$ | [24 → 24,21] |
| 45 | $((c(x)\|y \vee y)\|x \vee y) = c(x)$ | [22 → 24,33] |
| 47 | $((x\|c(x))\|c(x)) = x$ | [17 → 24] |
| 49 | $(x\|c(y)) \wedge (x\|y) = c(x)$ | [24 → 20] |
| 52,51 | $c(x \vee x) = c(x) \vee c(x)$ | [32 → 32] |
| 54,53 | $(x\|y) \vee (x\|y) = c(x \wedge y)$ | [20 → 32] |
| 55 | $((x\|y \vee y)\|(x\|c(y))) = x$ | [32 → 24] |
| 58,57 | $(c(x)\|y \vee y) = x \vee c(y)$ | [32 → 22] |
| 60,59 | $(x \vee x\|c(y)) = c(x) \vee y$ | [32 → 22] |
| 62,61 | $(x \vee c(y)\|x \vee y) = c(x)$ | [45:58] |
| 63 | $(((x\|y)\|x \wedge y)\|x \wedge y) = (x\|y)$ | [20 → 47,21] |
| 69 | $(x\|c(x)) \wedge c(x) = c(x)$ | [47 → 20] |
| 71 | $(x\|y \vee y) = (y\|x \wedge y)$ | [35 → 26] |
| 72 | $(x\|(x \wedge y) \wedge y) = c(x \wedge y)$ | [17 → 26] |
| 74 | $(x\|y \wedge x) = (y\|x \vee x)$ | [71] |
| 75 | $((x\|c(y \wedge z))\|(y\|x \wedge z)) = x$ | [26 → 24] |
| 77 | $x \wedge (y \wedge z) = y \wedge (x \wedge z)$ | [26 → 20,21] |
| 79,78 | $(x \vee x) \wedge c(y) = c(c(x) \vee y)$ | [32 → 37] |
| 82,81 | $c(x) \wedge (y \vee y) = c(x \vee c(y))$ | [32 → 37] |
| 94 | $((x\|y)\|x \wedge y) \wedge (x \wedge y) = x \wedge y$ | [20 → 69,21,21] |
| 96 | $(x\|c(y)) = ((y\|c(y))\|x \wedge c(y))$ | [69 → 26] |
| 102 | $(x \wedge y\|z \wedge u) = (x\|y) \vee (z\|u)$ | [20 → 29] |
| 118 | $(x \vee y) \wedge (c(x)\|y) = x \vee x$ | [41 → 20,33] |
| 122 | $(x\|y \vee y) \wedge (x\|c(y)) = c(x)$ | [32 → 49] |
| 132 | $(x \vee y\|c(x) \wedge y) \wedge c(x) = c(x \vee y)$ | [41 → 49,21] |
| 141,140 | $(x \vee c(y)) \wedge (x \vee y) = x \vee x$ | [22 → 49,33,58,33] |
| 147,146 | $(x \vee x\|y \vee y) = c(x) \vee c(y)$ | [32 → 57] |
| 155 | $(x \vee x\|y \wedge z) = c(x) \vee (y\|z)$ | [32 → 31] |
| 157,156 | $(x\|c(y) \wedge z) = y \vee (x\|z)$ | [26 → 31] |
| 161,160 | $(x \vee (x \vee y\|y)) \wedge c(x) = c(x \vee y)$ | [132:157] |
| 176 | $(x\|(x \vee c(x)) \wedge x) = c(x)$ | [61 → 71] |
| 184 | $x \wedge (y \wedge x) = y \wedge (x \vee x)$ | [71 → 20,21] |
| 185 | $x \wedge (y \vee y) = y \wedge (x \wedge y)$ | [184] |

| | | |
|---|---|---|
| 200 | $x \vee (y \vee x) = y \vee (x \vee x)$ | [37 → 74,23,58,33] |
| 203 | $x \vee (y \vee y) = y \vee (x \vee y)$ | [200] |
| 214 | $((x\|c(y \wedge x))\|(y\|x \vee x)) = x$ | [74 → 24] |
| 218 | $x \wedge (y \wedge (z \wedge u)) = z \wedge (x \wedge (y \wedge u))$ | [77 → 77] |
| 223 | $x \wedge c(y \vee z) = c(y) \wedge (x \wedge c(z))$ | [37 → 77] |
| 226 | $x \wedge (y \wedge (z \wedge u)) = y \wedge (z \wedge (x \wedge u))$ | [218] |
| 246,245 | $x \wedge ((x \vee c(x)) \wedge x) = x \vee x$ | [176 → 20,33] |
| 250,249 | $(c(x)\|(x\|(x \vee c(x)\|x))) = x$ | [176 → 39] |
| 251 | $((x\|y \vee y)\|(y\|(x\|y))) = y$ | [74 → 39] |
| 253 | $((x\|y \wedge z)\|(y\|(x\|z))) = y$ | [26 → 39] |
| 279 | $(x \wedge y\|x \wedge (y \vee y)) = (y\|(x \wedge y) \vee (x \wedge y))$ | [184 → 74] |
| 331,330 | $(x \vee (y \vee x)\|y \vee c(x)) = c(y)$ | [203 → 41,58] |
| 338,337 | $(x\|y \wedge x) \vee (y\|x \vee x) = c(y \wedge (x \vee x))$ | [71 → 53] |
| 342 | $c(x \wedge (y \vee y)) = c(y \wedge (x \wedge y))$ | [74 → 53,338] |
| 348 | $((x\|y \wedge x)\|(y\|c(x))) = y$ | [71 → 55] |
| 357,356 | $((x\|(x\|y) \wedge y)\|x \wedge y) = (x\|y)$ | [26 → 63] |
| 367,366 | $(x \vee y) \vee (x \vee y) = x \vee ((x \vee y) \vee y)$ | [37 → 72,38,23,38,33] |
| 373,372 | $x \vee (c(x) \wedge y\|y) = c(c(x) \wedge y)$ | [31 → 72] |
| 391,390 | $(x\|c((x \wedge y) \wedge y)) \wedge c(x \wedge y) = c(x)$ | [72 → 49] |
| 392 | $((x\|c((x \wedge y) \wedge y))\|c(x \wedge y)) = x$ | [72 → 24] |
| 400 | $(x \vee x) \wedge (y \wedge z) = c(c(x) \vee (y\|z))$ | [20 → 78] |
| 401 | $c(c(x) \vee (y\|z)) = (x \vee x) \wedge (y \wedge z)$ | [400] |
| 409 | $x \wedge (c(y) \wedge x) = c(y \vee c(x))$ | [185 → 81] |
| 420 | $(x \vee (y \vee x)) \wedge (y \vee c(x)) = y \vee y$ | [203 → 118,58] |
| 434 | $((c(x)\|y)\|x \vee x) = (x \vee y\|c(c(x) \wedge y))$ | [118 → 74,54] |
| 466,465 | $(x\|c(y \vee z)) = y \vee (x\|c(z))$ | [37 → 156] |
| 468,467 | $x \vee (x \vee ((c(x)\|y)\|y)\|y) = (c(x)\|y)$ | [63 → 156,157] |
| 469 | $x \vee (c(y)\|z) = y \vee (c(x)\|z)$ | [31 → 156] |
| 479 | $(x \vee (y\|z)\|(y\|(c(x)\|z))) = y$ | [156 → 39] |
| 486 | $c(x \vee (y\|z)) = y \wedge (c(x) \wedge z)$ | [156 → 20] |
| 488 | $x \wedge (c(y) \wedge z) = c(y \vee (x\|z))$ | [486] |
| 515 | $(x\|y \wedge x) \wedge (y\|c(x)) = c(y)$ | [348 → 20] |
| 548 | $((x\|y \vee y)\|((y\|c(y))\|x \wedge c(y))) = x$ | [69 → 75,33] |
| 641,640 | $((x\|c(y))\|c(x)) = ((x\|y \vee y)\|c(x \wedge c(y)))$ | [122 → 74,54] |
| 648,647 | $((x\|x \vee x)\|c(x \wedge c(x))) = x$ | [47:641] |
| 662 | $(x\|(x\|y) \wedge y) \wedge (x \wedge y) = x \wedge y$ | [26 → 94] |
| 684 | $c(x) \wedge (x\|(x \vee c(x)\|x)) = c(x)$ | [249 → 20] |
| 688 | $(x \vee c(y)\|(y\|(c(x)\|y))) = y$ | [57 → 251] |
| 700 | $(x\|y \vee y) \wedge (y\|(x\|y)) = c(y)$ | [251 → 20] |
| 704 | $x \vee ((x\|c(x))\|c(y)) = x \vee y$ | [43 → 96,23,23,157,161,466] |
| 790 | $((c(x)\|(x \vee y) \wedge y)\|c(x)) = x \vee y$ | [41 → 253] |
| 803,802 | $(x\|y \wedge z) \wedge (y\|(x\|z)) = c(y)$ | [253 → 20] |
| 845 | $(c(x) \vee (y \vee c(x))\|y \vee (x \vee x)) = c(y)$ | [32 → 330] |
| 868 | $(x \wedge (y \wedge x)) \vee z = (y \wedge (x \vee x)) \vee z$ | [342 → 22,23] |
| 927 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [22 → 465,23] |
| 928 | $\$Ans(AJ)$ | [927,1] |
| 1154 | $x \vee ((x\|c(x))\|y \vee y) = x \vee c(y)$ | [32 → 704] |
| 1156 | $x \vee ((x\|c(x))\|y \wedge z) = x \vee (y\|z)$ | [20 → 704] |
| 1163 | $x \vee (y\|z) = x \vee ((x\|c(x))\|y \wedge z)$ | [1156] |
| 1166 | $(x\|c(x)) \wedge c(x \vee y) = c(x \vee y)$ | [704 → 486,38] |
| 1241,1240 | $(x\|y \vee ((y \vee y) \vee y)) = c(y) \vee (x\|y \vee y)$ | [74 → 155,367] |
| 1291 | $((x\|c(c(y) \wedge x))\|y \vee c(x)) = x$ | [57 → 214] |
| 1384 | $(c(x) \vee (y \vee c(x))) \wedge (y \vee (x \vee x)) = y \vee y$ | [32 → 420] |
| 1456,1455 | $c(x) \wedge (y \wedge (x\|(x \vee c(x)\|x))) = y \wedge c(x)$ | [684 → 77] |

| | | |
|---|---|---|
| 1477 | $(c(x) \vee c(y)) \wedge (y\|(x \vee x\|y)) = c(y)$ | [146 → 700] |
| 2081 | $((x\|y) \vee (x \wedge y)\|c(x)) = x$ | [96 → 392,30,391] |
| 2100 | $(x \vee c(x)\|y \wedge (x \wedge z)) = (y\|x \wedge z)$ | [253 → 2081,803,21] |
| 2105 | $(c(x) \vee (x \vee x)\|c(x)) = x$ | [176 → 2081,246] |
| 2119 | $((x\|y \vee y) \vee (y \wedge (x \wedge y))\|c(y)) = y$ | [74 → 2081] |
| 2137 | $((x\|y \wedge z) \vee (y \wedge (x \wedge z))\|c(y)) = y$ | [26 → 2081] |
| 2198,2197 | $x \vee ((c(x) \wedge y) \vee c(c(x) \wedge y)\|y) = (c(x)\|y)$ | [469 → 2105,21,21,373,21,157] |
| 2648 | $c(x \vee (y \vee x)) \vee z = c(y \vee (x \vee x)) \vee z$ | [37 → 868,38,82,33] |
| 2652 | $c(x \vee (y \vee y)) \vee z = c(y \vee (x \vee y)) \vee z$ | [2648] |
| 2817 | $x \vee (y\|(x\|c(x)) \wedge y) = x \vee c(y)$ | [71 → 1154] |
| 2935 | $(x\|y) \vee (x\|y \vee y) = (y\|(x \wedge y) \vee (x \wedge y))$ | [102 → 279] |
| 2936 | $(x\|(y \wedge x) \vee (y \wedge x)) = (y\|x) \vee (y\|x \vee x)$ | [2935] |
| 2981 | $(x \vee x) \wedge (y \wedge ((x \vee c(x)) \wedge y)) = c(c(x) \vee c(y))$ | [2817 → 401,33,58] |
| 3112,3111 | $x \vee (y \vee ((x\|c(x))\|z)) = y \vee (c(x)\|z)$ | [469 → 1163,157] |
| 3169,3168 | $(x \vee c(x)\|x \wedge y) = (x\|y)$ | [662 → 2100,357] |
| 3201,3200 | $(x\|(x \vee c(x)) \wedge y) = (x\|y)$ | [26 → 3168] |
| 3212 | $(x\|y) \wedge (x\|(x \vee c(x)\|y)) = c(x)$ | [3168 → 802] |
| 3217,3216 | $((x\|y)\|(x\|(x \vee c(x)\|y))) = x$ | [3168 → 253] |
| 3228 | $(x \vee c(x)) \wedge (x \wedge y) = x \wedge y$ | [3168 → 20,21] |
| 3264 | $x \wedge ((x \vee c(x)) \wedge y) = x \wedge y$ | [3200 → 20,21] |
| 3283,3282 | $x \wedge (y \wedge ((x \vee c(x)) \wedge z)) = x \wedge (y \wedge z)$ | [226 → 3228] |
| 3290 | $((x \vee c(x)) \vee c(x \vee c(x))\|x \wedge y) = (x\|y)$ | [3228 → 3168,3169] |
| 3298 | $(x\|c(x)) \wedge (c(x) \wedge y) = c(x) \wedge y$ | [467 → 1166,21,468,21] |
| 3306 | $c(x) \wedge (c(x)\|c(x) \vee x) = c(x \vee c(x))$ | [1477 → 3264,33,82,60] |
| 3312 | $(x\|((x \vee c(x)) \vee c(x \vee c(x))) \wedge y) = (x\|y)$ | [3264 → 3200,3201] |
| 3390,3389 | $c(x \vee ((x\|c(x))\|y)) = c(x) \wedge y$ | [488 → 3298] |
| 3406,3405 | $(x \vee ((x\|c(x))\|y)) \vee z = (c(x)\|y) \vee z$ | [3298 → 29,30,157] |
| 3430 | $x \vee ((x\|c(x))\|y) = (c(x)\|y)$ | [3389 → 2105,3390,367,3406,3112,54,157,2198] |
| 3432 | $c(x) \vee (x \vee c(x)\|y) = (x \vee x\|y)$ | [32 → 3430,58,33] |
| 3538 | $(x \vee x\|(x\|(c(x)\|x))) = c(x) \vee x$ | [688 → 3432] |
| 3580 | $(x \vee (x \vee x)\|(x \vee x) \vee c(x)) = c(x) \vee c(x)$ | [3538 → 479,33,60,62,33,33] |
| 3593,3592 | $((x\|c(y)) \vee (x \wedge c(y))\|y \vee y) = c(y)$ | [684 → 2137,1456,33] |
| 3650 | $(c(x)\|x) = (x\|c(x)) \vee (x \wedge c(x))$ | [3200 → 548,21,3593,641,648,21] |
| 3663,3662 | $(x \vee c(x)) \vee c(x \vee c(x)) = c(x) \vee x$ | [32 → 3650,60,33,58,33,82] |
| 3664 | $(x\|(c(x) \vee x) \wedge y) = (x\|y)$ | [3312:3663] |
| 3668 | $(c(x) \vee x\|x \wedge y) = (x\|y)$ | [3290:3663] |
| 3683 | $c(x) \vee (c(x) \wedge x) = c(x)$ | [3650 → 434,3593,60] |
| 3692 | $(x \vee x) \vee c(c(x) \vee x) = x \vee x$ | [32 → 3683,33,79,33] |
| 3748,3747 | $(c(x)\|((x \vee x) \vee c(x)) \wedge y) = (c(x)\|y)$ | [32 → 3664] |
| 3788,3787 | $(c(x) \vee x\|x \vee x) = c(x)$ | [74 → 3664,18] |
| 3809 | $c(x) \wedge (c(x) \vee x\|c(x)) = c(c(x) \vee x)$ | [3664 → 515,18] |
| 3871 | $(c(x)\|(x\|(c(x) \vee x\|x))) = x$ | [3787 → 251] |
| 3955 | $((x \wedge y) \vee (x\|y)\|c(x)) = x$ | [3212 → 3668,21,3217] |
| 3976,3975 | $((x \vee x) \vee c(x)\|c(x)) = x$ | [684 → 3668,33,250] |
| 4022 | $(((x \vee x) \vee c(x)\|x \vee x)\|x) = (x \vee x) \vee c(x)$ | [3975 → 55] |
| 4029,4028 | $(x\|((x \vee x) \vee c(x)\|x)) = (x \vee x) \vee c(x)$ | [3975 → 24] |
| 4115 | $((c(x) \wedge (y \wedge c(z))) \vee (x \vee (y\|c(z)))\|c(y)) = y$ | [223 → 3955,466] |
| 4192,4191 | $c(x) \vee x = (x \vee x) \vee c(x)$ | [3692 → 1291,52,141,52,33,33,367,1241,3788,147,33] |
| 4196,4195 | $c(x) \wedge x = c((x \vee x) \vee c(x))$ | [3809:4192,3976,4192] |
| 4214,4213 | $(c(x)\|(x \vee x) \vee c(x)) = x$ | [3871:4192,4029] |
| 4236,4235 | $((x \vee x) \vee c(x)\|x \vee x) = c(x)$ | [3787:4192] |
| 4252,4251 | $(x \vee c(x)) \vee c(x \vee c(x)) = (x \vee x) \vee c(x)$ | [3662:4192] |
| 4274,4273 | $c((x \vee x) \vee c(x)) = c(x \vee c(x))$ | [3306:4192,4214,4196] |
| 4300,4299 | $(c(x)\|x) = (x \vee x) \vee c(x)$ | [4022:4236] |

| | | |
|---|---|---|
| 4401 | $c(x) \wedge x = c(x \vee c(x))$ | [4195:4274] |
| 4446,4445 | $x \wedge c(x \vee c(x)) = c(x \vee c(x))$ | [4401 → 409] |
| 4501,4500 | $(x \vee x) \vee c(x) = x \vee c(x)$ | [4401 → 356,4300,3748,4300,466,33,4236,4300] |
| 4503,4502 | $(x \vee c(x)\|c(x)) = x$ | [4401 → 2119,58,4446,4252,4501] |
| 4512 | $(x \vee (c(x) \vee c(x))\|c(x)) = x \vee x$ | [4401 → 43,18,33,466,33,18,52,18,33] |
| 4556,4555 | $c(x) \vee x = x \vee c(x)$ | [4191:4501] |
| 4610,4609 | $c(x) \vee c(x) = c(x)$ | [3580:4501,331] |
| 4647,4646 | $x \vee x = x$ | [4512:4610,4503] |
| 4833,4832 | $x \wedge y = c(c(x) \vee c(y))$ | [2981:4647,3283,36,4647] |
| 4834 | $\$Ans(DM)$ | [4832,2] |
| 4845,4844 | $(x\|y) = c(x) \vee c(y)$ | [2936:4833,4833,4647,466,33,4647,18,4647,4647] |
| 4846 | $\$Ans(DEF_SS)$ | [4844,5] |
| 4850,4849 | $c(x \vee (y \vee x)) \vee z = c(y \vee x) \vee z$ | [2652:4647] |
| 4905 | $c(c(x \vee c(y)) \vee c(x \vee y)) = x$ | [1384:4647,4833,4850,4647] |
| 4907 | $\$Ans(CUT_rewritten)$ | [4905,4] |
| 4919,4918 | $c(x \vee c(y)) \vee c(x \vee y) = c(x)$ | [845:4647,4845,4850] |
| 4922 | $c(c(x) \vee y) \vee x = x$ | [640:4845,33,4647,4845,33,4647,4647,4845,4833,33,4647,33,4647,4845,4919,33,4647] |
| 4931,4930 | $c(c(x)) = x$ | [32:4647] |
| 4981 | $c((x \vee (c(y) \vee z)) \vee c(x \vee (c(y) \vee z))) \vee y = y$ | [4115:4833,4931,4833,4931,4931,4845,4931,4556,4845,4931] |
| 5019 | $c(x \vee (c(x \vee y) \vee c(y))) \vee x = x \vee y$ | [790:4833,4845,4931,4931,4845,4931] |
| 5074 | $x \vee (y \vee x) = y \vee x$ | [4646 → 927] |
| 5078 | $x \vee (c(x \vee y) \vee y) = (x \vee y) \vee c(x \vee y)$ | [927 → 4555] |
| 5088 | $x \vee (y \vee c(y)) = c(y) \vee (x \vee y)$ | [4555 → 927] |
| 5095 | $c(x) \vee (y \vee x) = y \vee (x \vee c(x))$ | [5088] |
| 5116 | $c(x \vee y) \vee c(x) = c(x)$ | [4930 → 4922] |
| 5122 | $c(x \vee (c(y) \vee z)) \vee y = y$ | [927 → 4922] |
| 5151 | $c(x \vee c(y)) \vee y = y$ | [5074 → 4922] |
| 5157,5156 | $c(x \vee y) \vee c(y) = c(y)$ | [4930 → 5151] |
| 5160 | $c(x \vee c(y)) \vee x = x \vee y$ | [5019:5157] |
| 5237 | $c(x \vee (y \vee c(z))) \vee z = z$ | [5074 → 5122] |
| 5257,5256 | $c(x \vee y) \vee x = x \vee c(y)$ | [4930 → 5160] |
| 5260,5259 | $c(x \vee y) \vee y = y \vee c(x \vee y)$ | [5156 → 5160,4931] |
| 5263 | $x \vee c(x \vee y) = x \vee c(y)$ | [5116 → 5160,4931,5257] |
| 5268,5267 | $(x \vee y) \vee c(x \vee y) = x \vee (y \vee c(x \vee y))$ | [5078:5260] |
| 5271 | $c(x \vee ((c(y) \vee z) \vee c(x \vee (c(y) \vee z)))) \vee y = y$ | [4981:5268] |
| 5301 | $x \vee c(y \vee c(x)) = x$ | [5237 → 5256] |
| 5303 | $x \vee c(c(x) \vee y) = x$ | [5122 → 5256] |
| 5329 | $c(x) \vee c(y \vee x) = c(x)$ | [4930 → 5301] |
| 5349,5348 | $x \vee (y \vee c(x)) = x \vee c(x)$ | [5301 → 5256,4556,4931] |
| 5366 | $(x \vee y) \vee y = x \vee y$ | [5156 → 5303,4931] |
| 5370 | $(x \vee y) \vee x = x \vee y$ | [5116 → 5303,4931] |
| 5417,5416 | $x \vee (y \vee c(x \vee y)) = (x \vee y) \vee c(y)$ | [5366 → 5256,4556,5268] |
| 5431 | $c((x \vee (c(y) \vee z)) \vee c(c(y) \vee z)) \vee y = y$ | [5271:5417] |
| 5435 | $(x \vee y) \vee c(x \vee y) = (x \vee y) \vee c(y)$ | [5267:5417] |
| 5499,5498 | $c(x) \vee (y \vee x) = x \vee c(x)$ | [5329 → 5256,4931,4931] |
| 5506,5505 | $x \vee (y \vee c(y)) = y \vee c(y)$ | [5095:5499] |
| 5547,5546 | $x \vee (y \vee c(x \vee z)) = y \vee (x \vee c(z))$ | [5263 → 927] |
| 5549,5548 | $(x \vee y) \vee c(y) = y \vee c(y)$ | [5416:5547,5349] |
| 5553,5552 | $(x \vee y) \vee c(x \vee y) = y \vee c(y)$ | [5435:5549] |
| 5557,5556 | $c(x \vee c(x)) \vee y = y$ | [5431:5549,5553] |
| 5572 | $x \vee c(y \vee c(y)) = x$ | [5556 → 5370,5557] |
| 5595 | $x \vee c(x) = y \vee c(y)$ | [5572 → 5263,4931,5506] |

| | | |
|---|---|---|
| 5596 | $\$Ans(ONE)$ | [5595,3] |

**Proof BA-SS-2**

| | | |
|---|---|---|
| 1 | $(A\|((B\|C)\|(B\|C))) \neq (B\|((A\|C)\|(A\|C)))$ \| $\$Ans(A_SS)$ | [] |
| 2 | $((A\|(B\|B))\|(A\|B)) \neq A$ \| $\$Ans(CUT_SS)$ | [] |
| 3 | $A \vee B \neq ((A\|A)\|(B\|B))$ \| $\$Ans(DEF_J)$ | [] |
| 4 | $A \wedge B \neq ((A\|B)\|(A\|B))$ \| $\$Ans(DEF_M)$ | [] |
| 5 | $c(A) \neq (A\|A)$ \| $\$Ans(DEF_C)$ | [] |
| 7 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [] |
| 9,8 | $x \wedge y = c(c(x) \vee c(y))$ | [] |
| 11 | $(x \vee c(y)) \wedge (x \vee y) = x$ | [] |
| 12 | $c(c(x \vee c(y)) \vee c(x \vee y)) = x$ | [11,9] |
| 14 | $(x\|y) = c(x) \vee c(y)$ | [] |
| 16,15 | $c(x) \vee c(y) = (x\|y)$ | [14] |
| 17 | $x \vee (x \wedge y) = x$ | [] |
| 18 | $x \vee c((x\|y)) = x$ | [17,9,16] |
| 21,20 | $c(c(x)) = x$ | [] |
| 22 | $c((x \vee c(y)\|x \vee y)) = x$ | [12:16] |
| 24 | $x \wedge y = c((x\|y))$ | [8:16] |
| 34 | $x \vee c(y) = (c(x)\|y)$ | [20 → 15] |
| 42 | $x \vee (y\|z) = c(y) \vee (x \vee c(z))$ | [15 → 7] |
| 43 | $c(x) \vee (y \vee c(z)) = y \vee (x\|z)$ | [42] |
| 84 | $c(x) = (x \vee c(y)\|x \vee y)$ | [22 → 20] |
| 90 | $(x \vee c(y)\|x \vee y) = c(x)$ | [84] |
| 94 | $x \vee y = (c(x)\|c(y))$ | [20 → 34] |
| 96 | $(c(x)\|(x\|y)) = x$ | [18 → 34] |
| 99 | $(c(x)\|c(y)) = x \vee y$ | [94] |
| 110,109 | $c(x) = (x\|x)$ | [96 → 96,21] |
| 111 | $\$Ans(DEF_C)$ | [109,5] |
| 122,121 | $x \vee y = ((x\|x)\|(y\|y))$ | [99:110,110] |
| 123 | $\$Ans(DEF_J)$ | [121,3] |
| 125,124 | $((x\|x)\|(x\|y)) = x$ | [96:110] |
| 131 | $(((x\|x)\|y)\|((x\|x)\|(y\|y))) = (x\|x)$ | [90:110,122,125,122,110] |
| 149 | $(x\|(((y\|y)\|z)\|((y\|y)\|z))) = ((y\|y)\|((x\|z)\|(x\|z)))$ | [43:110,110,122,125,122,125,122] |
| 154 | $x \wedge y = ((x\|y)\|(x\|y))$ | [24:110] |
| 156 | $\$Ans(DEF_M)$ | [154,4] |
| 161 | $((x\|y)\|(x\|(y\|y))) = x$ | [124 → 131,125,125] |
| 221 | $((x\|(y\|y))\|(x\|y)) = x$ | [124 → 161] |
| 223 | $\$Ans(CUT_SS)$ | [221,2] |
| 424 | $(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$ | [124 → 149,125,125] |
| 425 | $\$Ans(A_SS)$ | [424,1] |

The following two Mace2 jobs show that the $\mathcal{BA}$ 2-basis {A_SS, CUT_SS} is independent.

**Countermodel BA-SS-a**

```
f(f(x,f(y,y)),f(x,y)) = x.                        % CUT_SS
f(A,f(f(B,C),f(B,C))) != f(B,f(f(A,C),f(A,C))).  % denial of A_SS
```

C: 0 A: 0 B: 1

| f: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |

Table 28: BA-SS-a.out

**Countermodel BA-SS-b**

```
f(x,f(f(y,z),f(y,z))) = f(y,f(f(x,z),f(x,z))).   % A_SS
f(f(A,f(B,B)),f(A,B)) != A.                      % denial of CUT_SS
```

B: 0 A: 1

| f: | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |

Table 29: BA-SS-b.out

### 2.3 Finding and Proving the Multiequation Bases

See the primary paper [8].

### 2.4 Are There Simpler Multiequation Bases?

See the primary paper [8].

## 3 Single Axioms

See the primary paper [8].

### 3.1 Generating and Filtering Candidates

See the primary paper [8].

### 3.2 Finite Ortholattices

See the primary paper [8].

### 3.3 Collecting and Applying Filters

We list here five filter sets. The first four, non-OL.A-4, non-OL.B-9, non-OL.C-23, and non-OL.D-14, are all non-ortholattices, meaning that they can be used to eliminate any $\mathcal{OL}$, $\mathcal{OML}$, $\mathcal{MOL}$, or $\mathcal{BA}$ candidate.

The fifth set, non-MOL-OML, consists of nonmodular orthomodular lattices, which can be used to eliminate $\mathcal{MOL}$ or $\mathcal{BA}$ candidates.

**Filter Set non-OL.A-4**

| f: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 2 | 1 | 0 |
| 1 | 1 | 1 | 0 |
| 2 | 0 | 0 | 0 |

Table 30: Structure 1 of non-OL.A-4

| f: | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 5 | 2 | 4 | 1 | 3 | 0 |
| 1 | 3 | 3 | 0 | 0 | 3 | 0 |
| 2 | 1 | 0 | 1 | 1 | 0 | 0 |
| 3 | 4 | 0 | 4 | 4 | 0 | 0 |
| 4 | 2 | 2 | 0 | 0 | 2 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 31: Structure 2 of non-OL.A-4

| f: | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 5 | 3 | 1 | 4 | 2 | 0 |
| 1 | 2 | 3 | 0 | 0 | 2 | 0 |
| 2 | 4 | 0 | 1 | 4 | 0 | 0 |
| 3 | 1 | 0 | 1 | 4 | 0 | 0 |
| 4 | 3 | 3 | 0 | 0 | 2 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 32: Structure 3 of non-OL.A-4

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| 1 | 6 | 6 | 0 | 0 | 3 | 2 | 0 | 0 |
| 2 | 5 | 0 | 5 | 0 | 3 | 0 | 1 | 0 |
| 3 | 4 | 0 | 0 | 4 | 0 | 2 | 1 | 0 |
| 4 | 3 | 3 | 3 | 0 | 3 | 0 | 0 | 0 |
| 5 | 2 | 2 | 0 | 2 | 0 | 2 | 0 | 0 |
| 6 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 33: Structure 4 of non-OL.A-4

**Filter Set non-OL.B-9**

| f: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 2 | 1 | 0 |
| 1 | 1 | 1 | 0 |
| 2 | 0 | 0 | 0 |

Table 34: Structure 1 of non-OL.B-9

| f: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 1 | 0 | 3 | 2 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 3 | 0 | 1 | 0 |
| 3 | 2 | 0 | 0 | 1 |

Table 35: Structure 2 of non-OL.B-9

| f: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 1 | 1 | 3 | 2 |
| 1 | 1 | 0 | 0 | 0 |
| 2 | 3 | 0 | 3 | 0 |
| 3 | 2 | 0 | 0 | 2 |

Table 36: Structure 3 of non-OL.B-9

| f: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 3 | 2 | 1 | 0 |
| 1 | 2 | 2 | 0 | 0 |
| 2 | 1 | 0 | 1 | 1 |
| 3 | 0 | 0 | 0 | 0 |

Table 37: Structure 4 of non-OL.B-9

| f: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 2 | 0 | 2 |
| 1 | 0 | 2 | 0 | 2 |
| 2 | 1 | 3 | 1 | 3 |
| 3 | 1 | 3 | 1 | 3 |

Table 38: Structure 5 of non-OL.B-9

| f: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 3 | 2 | 1 | 0 |
| 1 | 2 | 2 | 0 | 0 |
| 2 | 1 | 0 | 1 | 0 |
| 3 | 0 | 0 | 1 | 0 |

Table 39: Structure 6 of non-OL.B-9

| f: | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 2 | 1 | 4 | 3 |
| 1 | 2 | 3 | 0 | 1 | 4 |
| 2 | 1 | 0 | 4 | 3 | 2 |
| 3 | 4 | 1 | 3 | 2 | 0 |
| 4 | 3 | 4 | 2 | 0 | 1 |

Table 40: Structure 7 of non-OL.B-9

| f: | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 5 | 2 | 4 | 1 | 3 | 0 |
| 1 | 3 | 3 | 0 | 0 | 3 | 0 |
| 2 | 1 | 0 | 1 | 1 | 0 | 0 |
| 3 | 4 | 0 | 4 | 4 | 0 | 0 |
| 4 | 2 | 2 | 0 | 0 | 2 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 41: Structure 8 of non-OL.B-9

| f: | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 5 | 4 | 3 | 2 | 1 | 0 |
| 1 | 4 | 4 | 3 | 2 | 0 | 0 |
| 2 | 3 | 3 | 3 | 0 | 1 | 0 |
| 3 | 2 | 2 | 0 | 2 | 1 | 0 |
| 4 | 1 | 0 | 1 | 1 | 1 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 42: Structure 9 of non-OL.B-9

**Filter Set non-OL.C-23**

| f: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 2 | 1 | 0 |
| 1 | 1 | 1 | 0 |
| 2 | 0 | 0 | 0 |

Table 43: Structure 1 of non-OL.C-23

| f: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 1 | 2 |
| 1 | 2 | 0 | 1 |
| 2 | 1 | 2 | 0 |

Table 44: Structure 2 of non-OL.C-23

| f: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 2 | 1 |
| 1 | 1 | 0 | 2 |
| 2 | 2 | 1 | 0 |

Table 45: Structure 3 of non-OL.C-23

| f: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 1 | 0 | 1 |
| 1 | 2 | 2 | 1 |
| 2 | 2 | 0 | 0 |

Table 46: Structure 4 of non-OL.C-23

| f: | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 1 | 2 | 2 |
| 1 | 0 | 2 | 0 |
| 2 | 1 | 1 | 0 |

Table 47: Structure 5 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 1 | 1 | 3 | 2 |
| 1 | 1 | 0 | 0 | 0 |
| 2 | 3 | 0 | 3 | 0 |
| 3 | 2 | 0 | 0 | 2 |

Table 48: Structure 6 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 2 | 3 | 0 | 1 |
| 1 | 2 | 3 | 0 | 1 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 1 | 1 | 1 | 1 |

Table 49: Structure 7 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 3 | 2 | 1 | 0 |
| 1 | 2 | 2 | 0 | 0 |
| 2 | 1 | 0 | 1 | 1 |
| 3 | 0 | 0 | 0 | 0 |

Table 50: Structure 8 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 2 | 0 | 2 |
| 1 | 0 | 2 | 0 | 2 |
| 2 | 1 | 3 | 1 | 3 |
| 3 | 1 | 3 | 1 | 3 |

Table 51: Structure 9 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 3 | 1 | 2 |
| 1 | 2 | 1 | 3 | 0 |
| 2 | 3 | 0 | 2 | 1 |
| 3 | 1 | 2 | 0 | 3 |

Table 52: Structure 10 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 2 | 3 | 4 | 1 |
| 1 | 3 | 1 | 4 | 2 | 0 |
| 2 | 4 | 0 | 2 | 1 | 3 |
| 3 | 1 | 4 | 0 | 3 | 2 |
| 4 | 2 | 3 | 1 | 0 | 4 |

Table 53: Structure 11 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 3 | 4 | 1 | 2 |
| 1 | 2 | 1 | 0 | 4 | 3 |
| 2 | 3 | 4 | 2 | 0 | 1 |
| 3 | 4 | 2 | 1 | 3 | 0 |
| 4 | 1 | 0 | 3 | 2 | 4 |

Table 54: Structure 12 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 2 | 4 | 1 | 3 |
| 1 | 1 | 3 | 0 | 2 | 4 |
| 2 | 2 | 4 | 1 | 3 | 0 |
| 3 | 3 | 0 | 2 | 4 | 1 |
| 4 | 4 | 1 | 3 | 0 | 2 |

Table 55: Structure 13 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 2 | 1 | 4 | 3 |
| 1 | 2 | 3 | 0 | 1 | 4 |
| 2 | 1 | 0 | 4 | 3 | 2 |
| 3 | 4 | 1 | 3 | 2 | 0 |
| 4 | 3 | 4 | 2 | 0 | 1 |

Table 56: Structure 14 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 5 | 3 | 4 | 1 | 2 | 0 |
| 1 | 3 | 3 | 4 | 0 | 0 | 0 |
| 2 | 4 | 3 | 4 | 0 | 0 | 0 |
| 3 | 1 | 0 | 0 | 1 | 2 | 0 |
| 4 | 2 | 0 | 0 | 1 | 2 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 57: Structure 15 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 5 | 3 | 4 | 1 | 2 | 0 |
| 1 | 3 | 3 | 3 | 0 | 0 | 0 |
| 2 | 4 | 4 | 4 | 0 | 0 | 0 |
| 3 | 1 | 0 | 0 | 1 | 1 | 0 |
| 4 | 2 | 0 | 0 | 2 | 2 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 58: Structure 16 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 5 | 4 | 3 | 2 | 1 | 0 |
| 1 | 4 | 4 | 3 | 2 | 0 | 0 |
| 2 | 3 | 3 | 3 | 0 | 1 | 0 |
| 3 | 2 | 2 | 0 | 2 | 1 | 0 |
| 4 | 1 | 0 | 1 | 1 | 1 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 59: Structure 17 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 5 | 2 | 4 | 1 | 3 | 0 |
| 1 | 3 | 3 | 0 | 0 | 3 | 0 |
| 2 | 1 | 0 | 1 | 1 | 0 | 0 |
| 3 | 4 | 0 | 4 | 4 | 0 | 0 |
| 4 | 2 | 2 | 0 | 0 | 2 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 60: Structure 18 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 2 | 1 | 4 | 3 | 6 | 5 |
| 1 | 3 | 5 | 4 | 0 | 2 | 1 | 6 |
| 2 | 4 | 3 | 6 | 1 | 0 | 5 | 2 |
| 3 | 5 | 1 | 3 | 2 | 6 | 0 | 4 |
| 4 | 6 | 4 | 2 | 5 | 1 | 3 | 0 |
| 5 | 2 | 6 | 0 | 3 | 5 | 4 | 1 |
| 6 | 1 | 0 | 5 | 6 | 4 | 2 | 3 |

Table 61: Structure 19 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 2 | 1 | 4 | 3 | 6 | 5 |
| 1 | 2 | 1 | 0 | 5 | 6 | 3 | 4 |
| 2 | 1 | 0 | 2 | 6 | 5 | 4 | 3 |
| 3 | 4 | 5 | 6 | 3 | 0 | 1 | 2 |
| 4 | 3 | 6 | 5 | 0 | 4 | 2 | 1 |
| 5 | 6 | 3 | 4 | 1 | 2 | 5 | 0 |
| 6 | 5 | 4 | 3 | 2 | 1 | 0 | 6 |

Table 62: Structure 20 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 7 | 3 | 6 | 1 | 5 | 4 | 2 | 0 |
| 1 | 3 | 3 | 3 | 0 | 5 | 4 | 2 | 0 |
| 2 | 6 | 3 | 6 | 1 | 1 | 1 | 0 | 0 |
| 3 | 1 | 0 | 1 | 1 | 1 | 1 | 2 | 0 |
| 4 | 5 | 5 | 1 | 1 | 5 | 0 | 0 | 0 |
| 5 | 4 | 4 | 1 | 1 | 0 | 4 | 0 | 0 |
| 6 | 2 | 2 | 0 | 2 | 0 | 0 | 2 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 63: Structure 21 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 3 | 3 | 5 | 6 | 6 | 9 | 3 | 6 | 3 | 5 | 6 | 9 |
| 1 | 3 | 4 | 6 | 8 | 6 | 6 | 4 | 6 | 3 | 6 | 8 | 6 |
| 2 | 5 | 6 | 5 | 6 | 6 | 6 | 5 | 6 | 6 | 5 | 6 | 6 |
| 3 | 6 | 8 | 6 | 0 | 1 | 0 | 0 | 6 | 1 | 0 | 8 | 6 |
| 4 | 6 | 6 | 6 | 1 | 1 | 6 | 1 | 6 | 1 | 6 | 6 | 6 |
| 5 | 9 | 6 | 6 | 0 | 6 | 2 | 2 | 6 | 6 | 0 | 6 | 9 |
| 6 | 3 | 4 | 5 | 0 | 1 | 2 | 7 | 6 | 10 | 11 | 8 | 9 |
| 7 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 8 | 3 | 3 | 6 | 1 | 1 | 6 | 10 | 6 | 10 | 6 | 6 | 6 |
| 9 | 5 | 6 | 5 | 0 | 6 | 0 | 11 | 6 | 6 | 11 | 6 | 6 |
| 10 | 6 | 8 | 6 | 8 | 6 | 6 | 8 | 6 | 6 | 6 | 8 | 6 |
| 11 | 9 | 6 | 6 | 6 | 6 | 9 | 9 | 6 | 6 | 6 | 6 | 9 |

Table 64: Structure 22 of non-OL.C-23

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 3 | 3 | 3 | 6 | 6 | 6 | 3 | 6 | 3 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 3 | 6 |
| 1 | 3 | 4 | 10 | 8 | 6 | 6 | 4 | 6 | 3 | 8 | 14 | 10 | 10 | 6 | 10 | 14 | 3 | 6 |
| 2 | 3 | 10 | 5 | 16 | 6 | 6 | 5 | 6 | 3 | 6 | 12 | 10 | 10 | 12 | 10 | 6 | 3 | 16 |
| 3 | 6 | 8 | 16 | 0 | 1 | 2 | 0 | 6 | 1 | 8 | 1 | 6 | 2 | 6 | 1 | 6 | 2 | 16 |
| 4 | 6 | 6 | 6 | 1 | 1 | 6 | 1 | 6 | 1 | 6 | 1 | 6 | 6 | 6 | 1 | 6 | 6 | 6 |
| 5 | 6 | 6 | 6 | 2 | 6 | 2 | 2 | 6 | 6 | 6 | 2 | 6 | 2 | 6 | 6 | 6 | 2 | 6 |
| 6 | 3 | 4 | 5 | 0 | 1 | 2 | 7 | 6 | 9 | 8 | 11 | 10 | 13 | 12 | 15 | 14 | 17 | 16 |
| 7 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 8 | 3 | 3 | 3 | 1 | 1 | 6 | 9 | 6 | 9 | 6 | 1 | 6 | 6 | 6 | 1 | 6 | 3 | 6 |
| 9 | 6 | 8 | 6 | 8 | 6 | 6 | 8 | 6 | 6 | 8 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 10 | 6 | 14 | 12 | 1 | 1 | 2 | 11 | 6 | 1 | 6 | 11 | 6 | 2 | 12 | 1 | 14 | 2 | 6 |
| 11 | 6 | 10 | 10 | 6 | 6 | 6 | 10 | 6 | 6 | 6 | 6 | 10 | 10 | 6 | 10 | 6 | 6 | 6 |
| 12 | 6 | 10 | 10 | 2 | 6 | 2 | 13 | 6 | 6 | 6 | 2 | 10 | 13 | 6 | 10 | 6 | 2 | 6 |
| 13 | 6 | 6 | 12 | 6 | 6 | 6 | 12 | 6 | 6 | 6 | 12 | 6 | 6 | 12 | 6 | 6 | 6 | 6 |
| 14 | 6 | 10 | 10 | 1 | 1 | 6 | 15 | 6 | 1 | 6 | 1 | 10 | 10 | 6 | 15 | 6 | 6 | 6 |
| 15 | 6 | 14 | 6 | 6 | 6 | 6 | 14 | 6 | 6 | 6 | 14 | 6 | 6 | 6 | 6 | 14 | 6 | 6 |
| 16 | 3 | 3 | 3 | 2 | 6 | 2 | 17 | 6 | 3 | 6 | 2 | 6 | 2 | 6 | 6 | 6 | 17 | 6 |
| 17 | 6 | 6 | 16 | 16 | 6 | 6 | 16 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 16 |

Table 65: Structure 23 of non-OL.C-23

**Filter Set non-OL.D-14**

| f: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 |
| 1 | 2 | 3 | 0 | 1 |
| 2 | 3 | 2 | 1 | 0 |
| 3 | 1 | 0 | 3 | 2 |

Table 66: Structure 1 of non-OL.D-14

| f: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 3 | 2 |
| 1 | 2 | 3 | 1 | 0 |
| 2 | 0 | 1 | 3 | 2 |
| 3 | 2 | 3 | 1 | 0 |

Table 67: Structure 2 of non-OL.D-14

| f: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 1 | 2 | 2 | 3 | 3 |
| 2 | 1 | 1 | 0 | 0 |
| 3 | 3 | 3 | 2 | 2 |

Table 68: Structure 3 of non-OL.D-14

| f: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 2 | 3 | 1 |
| 1 | 1 | 3 | 2 | 0 |
| 2 | 2 | 0 | 1 | 3 |
| 3 | 3 | 1 | 0 | 2 |

Table 69: Structure 4 of non-OL.D-14

| f: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 2 | 2 | 0 | 1 |
| 1 | 3 | 3 | 0 | 1 |
| 2 | 0 | 0 | 0 | 1 |
| 3 | 1 | 1 | 0 | 1 |

Table 70: Structure 5 of non-OL.D-14

| f: | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 2 | 3 | 4 | 1 |
| 1 | 4 | 0 | 2 | 1 | 3 |
| 2 | 1 | 4 | 0 | 3 | 2 |
| 3 | 2 | 3 | 1 | 0 | 4 |
| 4 | 3 | 1 | 4 | 2 | 0 |

Table 71: Structure 6 of non-OL.D-14

| f: | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 2 | 3 | 4 | 1 |
| 1 | 1 | 3 | 0 | 2 | 4 |
| 2 | 2 | 1 | 4 | 0 | 3 |
| 3 | 3 | 4 | 2 | 1 | 0 |
| 4 | 4 | 0 | 1 | 3 | 2 |

Table 72: Structure 7 of non-OL.D-14

| f: | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 2 | 3 | 4 | 1 |
| 1 | 3 | 4 | 2 | 1 | 0 |
| 2 | 4 | 0 | 1 | 3 | 2 |
| 3 | 1 | 3 | 0 | 2 | 4 |
| 4 | 2 | 1 | 4 | 0 | 3 |

Table 73: Structure 8 of non-OL.D-14

| f: | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 |
| 1 | 2 | 3 | 4 | 0 | 1 |
| 2 | 4 | 0 | 1 | 2 | 3 |
| 3 | 1 | 2 | 3 | 4 | 0 |
| 4 | 3 | 4 | 0 | 1 | 2 |

Table 74: Structure 9 of non-OL.D-14

| f: | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 |
| 1 | 2 | 3 | 1 | 4 | 0 |
| 2 | 3 | 0 | 4 | 2 | 1 |
| 3 | 4 | 2 | 0 | 1 | 3 |
| 4 | 1 | 4 | 3 | 0 | 2 |

Table 75: Structure 10 of non-OL.D-14

| f: | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 2 | 3 | 4 | 1 |
| 1 | 4 | 0 | 1 | 3 | 2 |
| 2 | 1 | 3 | 0 | 2 | 4 |
| 3 | 2 | 1 | 4 | 0 | 3 |
| 4 | 3 | 4 | 2 | 1 | 0 |

Table 76: Structure 11 of non-OL.D-14

| f: | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 2 | 1 | 4 | 3 |
| 1 | 3 | 4 | 2 | 0 | 1 |
| 2 | 4 | 1 | 3 | 2 | 0 |
| 3 | 2 | 3 | 0 | 1 | 4 |
| 4 | 1 | 0 | 4 | 3 | 2 |

Table 77: Structure 12 of non-OL.D-14

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 2 | 3 | 4 | 5 | 6 | 1 |
| 1 | 6 | 1 | 0 | 5 | 3 | 2 | 4 |
| 2 | 1 | 5 | 2 | 0 | 6 | 4 | 3 |
| 3 | 2 | 4 | 6 | 3 | 0 | 1 | 5 |
| 4 | 3 | 6 | 5 | 1 | 4 | 0 | 2 |
| 5 | 4 | 3 | 1 | 6 | 2 | 5 | 0 |
| 6 | 5 | 0 | 4 | 2 | 1 | 3 | 6 |

Table 78: Structure 13 of non-OL.D-14

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 2 | 1 | 4 | 3 | 6 | 5 |
| 1 | 3 | 1 | 5 | 0 | 6 | 2 | 4 |
| 2 | 4 | 6 | 2 | 5 | 0 | 3 | 1 |
| 3 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| 4 | 5 | 3 | 6 | 1 | 4 | 0 | 2 |
| 5 | 2 | 4 | 0 | 6 | 1 | 5 | 3 |
| 6 | 1 | 0 | 3 | 2 | 5 | 4 | 6 |

Table 79: Structure 14 of non-OL.D-14

**Filter Set non-MOL-OML**

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 5 | 6 | 7 | 2 | 3 | 4 | 9 | 8 |
| 2 | 1 | 5 | 5 | 1 | 8 | 1 | 1 | 4 | 4 | 8 |
| 3 | 1 | 6 | 1 | 6 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 1 | 7 | 8 | 1 | 7 | 2 | 1 | 1 | 2 | 8 |
| 5 | 1 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 1 |
| 6 | 1 | 3 | 1 | 1 | 1 | 1 | 3 | 1 | 1 | 1 |
| 7 | 1 | 4 | 4 | 1 | 1 | 1 | 1 | 4 | 4 | 1 |
| 8 | 1 | 9 | 4 | 1 | 2 | 2 | 1 | 4 | 9 | 1 |
| 9 | 1 | 8 | 8 | 1 | 8 | 1 | 1 | 1 | 1 | 8 |

Table 80: Structure 1 of non-MOL-OML

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 5 | 6 | 7 | 2 | 3 | 4 | 9 | 8 | 11 | 10 |
| 2 | 1 | 5 | 5 | 1 | 8 | 1 | 1 | 4 | 4 | 8 | 1 | 1 |
| 3 | 1 | 6 | 1 | 6 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 1 | 7 | 8 | 1 | 7 | 2 | 1 | 1 | 2 | 8 | 1 | 1 |
| 5 | 1 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 1 | 1 | 1 |
| 6 | 1 | 3 | 1 | 1 | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 1 |
| 7 | 1 | 4 | 4 | 1 | 1 | 1 | 1 | 4 | 4 | 1 | 1 | 1 |
| 8 | 1 | 9 | 4 | 1 | 2 | 2 | 1 | 4 | 9 | 1 | 1 | 1 |
| 9 | 1 | 8 | 8 | 1 | 8 | 1 | 1 | 1 | 1 | 8 | 1 | 1 |
| 10 | 1 | 11 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 11 | 1 |
| 11 | 1 | 10 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 10 |

Table 81: Structure 2 of non-MOL-OML

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 5 | 6 | 7 | 2 | 3 | 4 | 9 | 8 | 11 | 10 | 13 | 12 |
| 2 | 1 | 5 | 5 | 1 | 8 | 1 | 1 | 1 | 10 | 8 | 8 | 10 | 8 | 1 |
| 3 | 1 | 6 | 1 | 6 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 1 | 7 | 8 | 1 | 7 | 1 | 1 | 1 | 12 | 8 | 8 | 1 | 8 | 12 |
| 5 | 1 | 2 | 1 | 1 | 1 | 2 | 1 | 1 | 2 | 1 | 2 | 1 | 1 | 1 |
| 6 | 1 | 3 | 1 | 1 | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 7 | 1 | 4 | 1 | 1 | 1 | 1 | 1 | 4 | 4 | 1 | 1 | 1 | 4 | 1 |
| 8 | 1 | 9 | 10 | 1 | 12 | 2 | 1 | 4 | 9 | 1 | 2 | 10 | 4 | 12 |
| 9 | 1 | 8 | 8 | 1 | 8 | 1 | 1 | 1 | 1 | 8 | 8 | 1 | 8 | 1 |
| 10 | 1 | 11 | 8 | 1 | 8 | 2 | 1 | 1 | 2 | 8 | 11 | 1 | 8 | 1 |
| 11 | 1 | 10 | 10 | 1 | 1 | 1 | 1 | 1 | 10 | 1 | 1 | 10 | 1 | 1 |
| 12 | 1 | 13 | 8 | 1 | 8 | 1 | 1 | 4 | 4 | 8 | 8 | 1 | 13 | 1 |
| 13 | 1 | 12 | 1 | 1 | 12 | 1 | 1 | 1 | 12 | 1 | 1 | 1 | 1 | 12 |

Table 82: Structure 3 of non-MOL-OML

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 5 | 6 | 7 | 2 | 3 | 4 | 9 | 8 | 11 | 10 | 13 | 12 |
| 2 | 1 | 5 | 5 | 1 | 8 | 1 | 1 | 4 | 4 | 8 | 1 | 1 | 1 | 1 |
| 3 | 1 | 6 | 1 | 6 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 1 | 7 | 8 | 1 | 7 | 2 | 1 | 1 | 2 | 8 | 1 | 1 | 1 | 1 |
| 5 | 1 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1 |
| 6 | 1 | 3 | 1 | 1 | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 7 | 1 | 4 | 4 | 1 | 1 | 1 | 1 | 4 | 4 | 1 | 1 | 1 | 1 | 1 |
| 8 | 1 | 9 | 4 | 1 | 2 | 2 | 1 | 4 | 9 | 1 | 1 | 1 | 1 | 1 |
| 9 | 1 | 8 | 8 | 1 | 8 | 1 | 1 | 1 | 1 | 8 | 1 | 1 | 1 | 1 |
| 10 | 1 | 11 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 11 | 1 | 1 | 1 |
| 11 | 1 | 10 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 10 | 1 | 1 |
| 12 | 1 | 13 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 13 | 1 |
| 13 | 1 | 12 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 12 |

Table 83: Structure 4 of non-MOL-OML

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 5 | 6 | 7 | 2 | 3 | 4 | 9 | 8 | 11 | 10 | 13 | 12 |
| 2 | 1 | 5 | 5 | 1 | 8 | 1 | 1 | 4 | 4 | 8 | 1 | 1 | 1 | 1 |
| 3 | 1 | 6 | 1 | 6 | 1 | 1 | 1 | 1 | 1 | 1 | 12 | 10 | 10 | 12 |
| 4 | 1 | 7 | 8 | 1 | 7 | 2 | 1 | 1 | 2 | 8 | 1 | 1 | 1 | 1 |
| 5 | 1 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1 |
| 6 | 1 | 3 | 1 | 1 | 1 | 1 | 3 | 1 | 1 | 1 | 3 | 1 | 3 | 1 |
| 7 | 1 | 4 | 4 | 1 | 1 | 1 | 1 | 4 | 4 | 1 | 1 | 1 | 1 | 1 |
| 8 | 1 | 9 | 4 | 1 | 2 | 2 | 1 | 4 | 9 | 1 | 1 | 1 | 1 | 1 |
| 9 | 1 | 8 | 8 | 1 | 8 | 1 | 1 | 1 | 1 | 8 | 1 | 1 | 1 | 1 |
| 10 | 1 | 11 | 1 | 12 | 1 | 1 | 3 | 1 | 1 | 1 | 11 | 1 | 3 | 12 |
| 11 | 1 | 10 | 1 | 10 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 10 | 10 | 1 |
| 12 | 1 | 13 | 1 | 10 | 1 | 1 | 3 | 1 | 1 | 1 | 3 | 10 | 13 | 1 |
| 13 | 1 | 12 | 1 | 12 | 1 | 1 | 1 | 1 | 1 | 1 | 12 | 1 | 1 | 12 |

Table 84: Structure 5 of non-MOL-OML

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 5 | 6 | 7 | 2 | 3 | 4 | 9 | 8 | 11 | 10 | 13 | 12 | 15 | 14 |
| 2 | 1 | 5 | 5 | 1 | 8 | 1 | 1 | 1 | 10 | 8 | 8 | 10 | 8 | 1 | 1 | 1 |
| 3 | 1 | 6 | 1 | 6 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 1 | 7 | 8 | 1 | 7 | 1 | 1 | 1 | 12 | 8 | 8 | 1 | 8 | 12 | 1 | 1 |
| 5 | 1 | 2 | 1 | 1 | 1 | 2 | 1 | 1 | 2 | 1 | 2 | 1 | 1 | 1 | 1 | 1 |
| 6 | 1 | 3 | 1 | 1 | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 7 | 1 | 4 | 1 | 1 | 1 | 1 | 1 | 4 | 4 | 1 | 1 | 1 | 4 | 1 | 1 | 1 |
| 8 | 1 | 9 | 10 | 1 | 12 | 2 | 1 | 4 | 9 | 1 | 2 | 10 | 4 | 12 | 1 | 1 |
| 9 | 1 | 8 | 8 | 1 | 8 | 1 | 1 | 1 | 1 | 8 | 8 | 1 | 8 | 1 | 1 | 1 |
| 10 | 1 | 11 | 8 | 1 | 8 | 2 | 1 | 1 | 2 | 8 | 11 | 1 | 8 | 1 | 1 | 1 |
| 11 | 1 | 10 | 10 | 1 | 1 | 1 | 1 | 1 | 10 | 1 | 1 | 10 | 1 | 1 | 1 | 1 |
| 12 | 1 | 13 | 8 | 1 | 8 | 1 | 1 | 4 | 4 | 8 | 8 | 1 | 13 | 1 | 1 | 1 |
| 13 | 1 | 12 | 1 | 1 | 12 | 1 | 1 | 1 | 12 | 1 | 1 | 1 | 1 | 12 | 1 | 1 |
| 14 | 1 | 15 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 15 | 1 |
| 15 | 1 | 14 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 14 |

Table 85: Structure 6 of non-MOL-OML

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 5 | 6 | 7 | 2 | 3 | 4 | 9 | 8 | 11 | 10 | 13 | 12 | 15 | 14 |
| 2 | 1 | 5 | 5 | 1 | 8 | 1 | 1 | 1 | 10 | 8 | 8 | 10 | 8 | 1 | 1 | 1 |
| 3 | 1 | 6 | 1 | 6 | 12 | 1 | 1 | 1 | 12 | 1 | 1 | 1 | 14 | 12 | 12 | 14 |
| 4 | 1 | 7 | 8 | 12 | 7 | 1 | 1 | 1 | 12 | 8 | 8 | 1 | 8 | 12 | 12 | 1 |
| 5 | 1 | 2 | 1 | 1 | 1 | 2 | 1 | 1 | 2 | 1 | 2 | 1 | 1 | 1 | 1 | 1 |
| 6 | 1 | 3 | 1 | 1 | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 3 | 1 | 3 | 1 |
| 7 | 1 | 4 | 1 | 1 | 1 | 1 | 1 | 4 | 4 | 1 | 1 | 1 | 4 | 1 | 1 | 1 |
| 8 | 1 | 9 | 10 | 12 | 12 | 2 | 1 | 4 | 9 | 1 | 2 | 10 | 4 | 12 | 12 | 1 |
| 9 | 1 | 8 | 8 | 1 | 8 | 1 | 1 | 1 | 1 | 8 | 8 | 1 | 8 | 1 | 1 | 1 |
| 10 | 1 | 11 | 8 | 1 | 8 | 2 | 1 | 1 | 2 | 8 | 11 | 1 | 8 | 1 | 1 | 1 |
| 11 | 1 | 10 | 10 | 1 | 1 | 1 | 1 | 1 | 10 | 1 | 1 | 10 | 1 | 1 | 1 | 1 |
| 12 | 1 | 13 | 8 | 14 | 8 | 1 | 3 | 4 | 4 | 8 | 8 | 1 | 13 | 1 | 3 | 14 |
| 13 | 1 | 12 | 1 | 12 | 12 | 1 | 1 | 1 | 12 | 1 | 1 | 1 | 1 | 12 | 12 | 1 |
| 14 | 1 | 15 | 1 | 12 | 12 | 1 | 3 | 1 | 12 | 1 | 1 | 1 | 3 | 12 | 15 | 1 |
| 15 | 1 | 14 | 1 | 14 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 14 | 1 | 1 | 14 |

Table 86: Structure 7 of non-MOL-OML

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 5 | 6 | 7 | 2 | 3 | 4 | 9 | 8 | 11 | 10 | 13 | 12 | 15 | 14 |
| 2 | 1 | 5 | 5 | 1 | 8 | 1 | 1 | 4 | 4 | 8 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 1 | 6 | 1 | 6 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 1 | 7 | 8 | 1 | 7 | 2 | 1 | 1 | 2 | 8 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 1 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | 1 | 3 | 1 | 1 | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 7 | 1 | 4 | 4 | 1 | 1 | 1 | 1 | 4 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 8 | 1 | 9 | 4 | 1 | 2 | 2 | 1 | 4 | 9 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 9 | 1 | 8 | 8 | 1 | 8 | 1 | 1 | 1 | 1 | 8 | 1 | 1 | 1 | 1 | 1 | 1 |
| 10 | 1 | 11 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 11 | 1 | 1 | 1 | 1 | 1 |
| 11 | 1 | 10 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 10 | 1 | 1 | 1 | 1 |
| 12 | 1 | 13 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 13 | 1 | 1 | 1 |
| 13 | 1 | 12 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 12 | 1 | 1 |
| 14 | 1 | 15 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 15 | 1 |
| 15 | 1 | 14 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 14 |

Table 87: Structure 8 of non-MOL-OML

| f: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 5 | 6 | 7 | 2 | 3 | 4 | 9 | 8 | 11 | 10 | 13 | 12 | 15 | 14 |
| 2 | 1 | 5 | 5 | 1 | 8 | 1 | 1 | 4 | 4 | 8 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 1 | 6 | 1 | 6 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 1 | 7 | 8 | 1 | 7 | 2 | 1 | 1 | 2 | 8 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 1 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | 1 | 3 | 1 | 1 | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 7 | 1 | 4 | 4 | 1 | 1 | 1 | 1 | 4 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 8 | 1 | 9 | 4 | 1 | 2 | 2 | 1 | 4 | 9 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 9 | 1 | 8 | 8 | 1 | 8 | 1 | 1 | 1 | 1 | 8 | 1 | 1 | 1 | 1 | 1 | 1 |
| 10 | 1 | 11 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 11 | 1 | 14 | 12 | 12 | 14 |
| 11 | 1 | 10 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 10 | 10 | 1 | 10 | 1 |
| 12 | 1 | 13 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 14 | 10 | 13 | 1 | 10 | 14 |
| 13 | 1 | 12 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 12 | 1 | 1 | 12 | 12 | 1 |
| 14 | 1 | 15 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 12 | 10 | 10 | 12 | 15 | 1 |
| 15 | 1 | 14 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 14 | 1 | 14 | 1 | 1 | 14 |

Table 88: Structure 9 of non-MOL-OML

## 3.4 Trying to Prove That Candidates Are Single Axioms

See the primary paper [8].

## 3.5 Single Axioms for $\mathcal{OL}$, $\mathcal{OML}$, and $\mathcal{MOL}$

The single axioms for the varieties under study are the following.

OL-Sh: $(((y \mid x) \mid (x \mid z)) \mid u) \mid (x \mid ((x \mid ((y \mid y)) \mid z)) = x$
OML-Sh: $(((y \mid x) \mid (x \mid z)) \mid u) \mid (x \mid ((z \mid ((x \mid x) \mid z)) \mid z)) = x$
MOL-Sh: $(y \mid x) \mid (((x \mid x) \mid z) \mid (((((x \mid y) \mid z) \mid z) \mid x) \mid (x \mid u))) = x$

For each, we prove that it is equivalent to the multiequation Sheffer stroke basis for that variety given in Section 2.2.

### 3.5.1 A Single Axiom for Ortholattices

We list in this subsection three Otter proofs:

1. OL-Sh implies the join/complement basis {AJ, B1, DM, CC, ONE}.
2. OL-Sh implies the Sheffer stroke basis {A_SS, B_SS, ONE_SS}.
3. The Sheffer stroke basis {A_SS, B_SS, ONE_SS} implies OL-Sh.

The first two prove essentially the same theorem—either one is sufficient. For the first proof we have rewritten the goal B1 to eliminate the meet operation; this allows a convenient forward proof.

**Proof OL-Sh-jc**

| | | |
|---|---|---|
| 1 | $A \vee (B \vee C) \neq B \vee (A \vee C) \mid \$Ans(AJ)$ | [] |
| 2 | $A \vee c(c(A) \vee B) \neq A \mid \$Ans(B1_rewritten)$ | [] |
| 3 | $A \wedge B \neq c(c(A) \vee c(B)) \mid \$Ans(DM)$ | [] |
| 4 | $c(c(A)) \neq A \mid \$Ans(CC)$ | [] |
| 5 | $A \vee c(A) \neq B \vee c(B) \mid \$Ans(ONE)$ | [] |
| 6 | $(A\|B) \neq c(A) \vee c(B) \mid \$Ans(DEF_SS)$ | [] |
| 8 | $((((x\|y)\|(y\|z))\|u)\|(y\|((y\|((x\|x)\|x))\|z))) = y$ | [] |
| 10 | $x \vee y = ((x\|x)\|(y\|y))$ | [] |
| 11 | $((x\|x)\|(y\|y)) = x \vee y$ | [10] |
| 13 | $x \wedge y = ((x\|y)\|(x\|y))$ | [] |
| 14 | $((x\|y)\|(x\|y)) = x \wedge y$ | [13] |
| 16 | $c(x) = (x\|x)$ | [] |
| 18,17 | $(x\|x) = c(x)$ | [16] |
| 20,19 | $c((x\|y)) = x \wedge y$ | [14:18] |
| 21 | $(c(x)\|c(y)) = x \vee y$ | [11:18,18] |
| 23 | $((((x\|y)\|(y\|z))\|u)\|(y\|((y\|(c(x)\|x))\|z))) = y$ | [8:18] |
| 31,30 | $c(c(x)) = x \vee x$ | [17 → 21] |
| 35 | $c(x) \wedge c(y) = c(x \vee y)$ | [21 → 19] |
| 38,37 | $c(x \vee x) = c(x) \vee c(x)$ | [30 → 30] |
| 41 | $(c(x)\|y \vee y) = x \vee c(y)$ | [30 → 21] |
| 44,43 | $(x \vee x\|c(y)) = c(x) \vee y$ | [30 → 21] |
| 45 | $(((x \vee y\|(c(y)\|z))\|u)\|(c(y)\|((c(y)\|c(x) \vee x)\|z))) = c(y)$ | [21 → 23,31,44] |
| 47 | $(((c(x)\|(x\|y))\|z)\|(x\|((x\|(c(x)\|x))\|y))) = x$ | [17 → 23] |
| 49 | $((((x\|c(y))\|y \vee z)\|u)\|(c(y)\|((c(y)\|(c(x)\|x))\|c(z)))) = c(y)$ | [21 → 23] |
| 53 | $((x\|y)\|(x\|((x\|((z\|x) \wedge (x\|u)\|((z\|x)\|(x\|u))))\|((x\|(c(z)\|z))\|u)))) = x$ | [23 → 23,20] |
| 55 | $((x \vee x\|y)\|(x\|((x\|(c(x)\|x))\|x))) = x$ | [17 → 23,18,31] |
| 71 | $(x \vee x) \wedge c(y) = c(c(x) \vee y)$ | [30 → 35] |
| 84,83 | $(x \vee x\|y \vee y) = c(x) \vee c(y)$ | [30 → 41] |
| 131 | $(((x \vee y\|y \vee z)\|u)\|(c(y)\|((c(y)\|c(x) \vee x)\|c(z)))) = c(y)$ | [21 → 45] |
| 139 | $(((x \vee y\|(c(y)\|(c(y)\|c(x) \vee x)))\|z)\|(c(y)\|c(y) \wedge (c(x) \vee x))) = c(y)$ | [17 → 45,20] |
| 151 | $(x \vee x) \wedge (y \vee y) = c(c(x) \vee c(y))$ | [30 → 71] |
| 259 | $(((c(x) \vee y\|y \vee z)\|u)\|(c(y)\|((c(y)\|(x \vee x) \vee c(x))\|c(z)))) = c(y)$ | [43 → 49,38,84,31] |
| 263 | $((x\|c(y)) \wedge (y \vee z)\|(c(y)\|((c(y)\|(c(x)\|x))\|c(z)))) = c(y)$ | [17 → 49,20] |
| 467 | $(c(x)\|(x\|((x\|(c(x)\|x))\|y))) = x$ | [53 → 47] |

| | | |
|---|---|---|
| 469 | $(x \vee y\|(c(y)\|((c(y)\|c(x) \vee x)\|z))) = c(y)$ | [53 → 45] |
| 526,525 | $x \vee x = x$ | [53 → 467,18,18,31] |
| 531 | $(c(x) \wedge (x\|y)\|((c(x)\|(x\|y))\|x)) = (c(x)\|(x\|y))$ | [47 → 467,20] |
| 661 | $(((c(x) \vee y\|y \vee z)\|u)\|(c(y)\|((c(y)\|x \vee c(x))\|c(z)))) = c(y)$ | [259:526] |
| 700,699 | $x \wedge y = c(c(x) \vee c(y))$ | [151:526,526] |
| 701 | $\$Ans(DM)$ | [699,3] |
| 725,724 | $(x\|y) = c(x) \vee c(y)$ | [83:526,526] |
| 726 | $\$Ans(DEF_SS)$ | [724,6] |
| 727 | $c(c(x) \vee c(y)) \vee c(c(x) \vee c(c(c(x) \vee c(x \vee c(x))) \vee c(x))) = x$ | [55:526,725,725,31,526,725,725,725,725] |
| 730,729 | $c(c(x)) = x$ | [30:526] |
| 731 | $\$Ans(CC)$ | [729,4] |
| 740 | $(x \vee c(c(x) \vee c(y))) \vee c(c(x \vee c(c(x) \vee c(y))) \vee c(x)) = x \vee c(c(x) \vee c(y))$ | [531:725,700,730,725,725,730,725,725,730,725,725,730] |
| 755,754 | $(c(c(x) \vee y) \vee c(y \vee z)) \vee c(y \vee c(c(y \vee c(x \vee c(x))) \vee z)) = c(y)$ | [263:725,730,700,725,730,725,730,725,730,725,730,725,730] |
| 762 | $c(c(c(x \vee y) \vee c(y \vee c(y \vee c(c(x) \vee x)))) \vee c(z)) \vee c(y \vee (y \vee c(c(x) \vee x))) = c(y)$ | [139:725,730,725,730,725,725,700,730,725,730,730,725] |
| 782 | $c(c(c(c(x) \vee y) \vee c(y \vee z)) \vee c(u)) \vee c(y \vee c(c(y \vee c(x \vee c(x))) \vee z)) = c(y)$ | [661:725,725,725,730,725,730,725,730,725] |
| 794 | $c(x \vee y) \vee c(y \vee c(c(y \vee c(c(x) \vee x)) \vee c(z))) = c(y)$ | [469:725,730,725,725,730,725] |
| 800 | $c(c(c(x \vee y) \vee c(y \vee z)) \vee c(u)) \vee c(y \vee c(c(y \vee c(c(x) \vee x)) \vee z)) = c(y)$ | [131:725,725,725,730,725,730,725,730,725] |
| 810 | $c(c(x) \vee y) \vee c(c(x) \vee c(c(c(x) \vee c(x \vee c(x))) \vee c(x))) = x$ | [729 → 727] |
| 813,812 | $c(c(x) \vee c(c(c(x) \vee c(x \vee c(x))) \vee c(x))) = x$ | [525 → 727] |
| 816 | $c(c(x) \vee y) \vee x = x$ | [810:813] |
| 819,818 | $c(x \vee y) \vee c(x) = c(x)$ | [729 → 816] |
| 822 | $(x \vee c(c(x) \vee c(y))) \vee x = x \vee c(c(x) \vee c(y))$ | [740:819,730] |
| 831,830 | $x \vee (x \vee y) = x \vee y$ | [818 → 818,730,730,730] |
| 834 | $c(c(c(x \vee y) \vee c(y \vee c(y \vee c(c(x) \vee x)))) \vee c(z)) \vee c(y \vee c(c(x) \vee x)) = c(y)$ | [762:831] |
| 871,870 | $(x \vee c(c(x) \vee y)) \vee x = x \vee c(c(x) \vee y)$ | [729 → 822,730] |
| 872 | $((x \vee y) \vee x) \vee (x \vee y) = (x \vee y) \vee x$ | [818 → 822,730,819,730] |
| 875,874 | $(c(x) \vee c(x \vee y)) \vee c(x) = c(x) \vee c(x \vee y)$ | [818 → 872,819,819] |
| 980 | $(c(c(x) \vee y) \vee c(y \vee z)) \vee c(y) = c(y)$ | [754 → 830,755] |
| 1030 | $c((x \vee y) \vee c(z)) \vee c(x \vee c(c(x \vee c(c(x \vee y) \vee (x \vee y))) \vee y)) = c(x)$ | [816 → 782,730,730] |
| 1060 | $(c(x \vee y) \vee c(y \vee z)) \vee c(y) = c(y)$ | [729 → 980] |
| 1072 | $x \vee c(c(x) \vee y) = x$ | [818 → 980,730,730,871,730] |
| 1074 | $\$Ans(B1_rewritten)$ | [1072,2] |
| 1075 | $c(x) \vee c(x \vee y) = c(x)$ | [816 → 980,875] |
| 1093 | $(x \vee y) \vee x = x \vee y$ | [818 → 1072,730] |
| 1143 | $(x \vee c(c(x \vee y) \vee z)) \vee (x \vee y) = x \vee y$ | [1075 → 1060,730,730,730] |
| 1151 | $(c(x \vee c(y \vee z)) \vee y) \vee (y \vee z) = y \vee z$ | [818 → 1060,730,730,730] |
| 1220,1219 | $c(x \vee y) \vee c(y) = c(y)$ | [818 → 794,730,526] |
| 1243 | $c(x \vee c(y)) \vee y = y$ | [729 → 1219,730] |
| 1253 | $x \vee (y \vee x) = y \vee x$ | [1219 → 816,730] |
| 1257 | $c(x) \vee c(y \vee x) = c(x)$ | [1219 → 1093,1220] |
| 1322,1321 | $c(x \vee c(c(x \vee c(c(y) \vee y)) \vee z)) = c(x)$ | [1243 → 800] |
| 1335 | $c((x \vee y) \vee c(z)) \vee c(x) = c(x)$ | [1030:1322] |
| 1359 | $c((x \vee y) \vee z) \vee c(x) = c(x)$ | [729 → 1335] |
| 1383 | $c((c(x) \vee y) \vee z) \vee x = x$ | [729 → 1359,730] |
| 1429 | $c(x \vee c(c(y) \vee y)) = c(x)$ | [1243 → 834] |
| 1439 | $c((x \vee c(y)) \vee z) \vee y = y$ | [1253 → 1383] |
| 1442,1441 | $c(x \vee (c(y) \vee z)) \vee y = y$ | [1253 → 1383] |
| 1453 | $x \vee (c(y) \vee y) = c(y) \vee y$ | [1243 → 1429,730,730,730] |

| | | |
|---|---|---|
| 1457 | $(c(x) \vee x) \vee y = c(x) \vee x$ | [818 → 1429,730,730] |
| 1532,1531 | $c(x \vee (y \vee c(z))) \vee z = z$ | [1253 → 1439] |
| 1583 | $x \vee c(y \vee (c(x) \vee z)) = x$ | [1441 → 1093,1442] |
| 1617 | $c(x) \vee x = c(y) \vee y$ | [1453 → 1457] |
| 1618 | $x \vee c(x) = c(y) \vee y$ | [729 → 1617] |
| 1619 | $c(x) \vee x = y \vee c(y)$ | [1618] |
| 1641,1640 | $(x \vee y) \vee (y \vee x) = y \vee x$ | [1257 → 1151,730] |
| 1656 | $x \vee c(x) = y \vee c(y)$ | [729 → 1619] |
| 1657 | $\$Ans(ONE)$ | [1656,5] |
| 1686 | $c(x \vee (y \vee z)) \vee c(z) = c(z)$ | [729 → 1531] |
| 1710 | $x \vee c(y \vee (z \vee c(x))) = x$ | [1531 → 1093,1532] |
| 1758 | $c(x) \vee c(y \vee (x \vee z)) = c(x)$ | [729 → 1583] |
| 1846 | $x \vee y = y \vee x$ | [1640 → 1093,1641,1641] |
| 1881 | $c(x) \vee c(y \vee (z \vee x)) = c(x)$ | [729 → 1710] |
| 2012,2011 | $(x \vee y) \vee (x \vee (z \vee y)) = x \vee (z \vee y)$ | [1686 → 1143,730] |
| 2081 | $(x \vee y) \vee (y \vee (x \vee z)) = y \vee (x \vee z)$ | [1758 → 1151,730] |
| 2089 | $(x \vee y) \vee (y \vee (z \vee x)) = y \vee (z \vee x)$ | [1881 → 1151,730] |
| 10276,10275 | $(x \vee (y \vee z)) \vee (y \vee x) = x \vee (y \vee z)$ | [1846 → 2081] |
| 10439 | $(x \vee y) \vee (z \vee y) = z \vee (x \vee y)$ | [2011 → 2089,10276,2012] |
| 10467 | $x \vee (y \vee z) = (y \vee z) \vee (x \vee z)$ | [10439] |
| 10981,10980 | $(x \vee y) \vee (z \vee y) = x \vee (z \vee y)$ | [1846 → 10439] |
| 11000 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [10467:10981] |
| 11001 | $\$Ans(AJ)$ | [11000,1] |

**Proof OL-Sh**

| | | |
|---|---|---|
| 1 | $(A\|((B\|C)\|(B\|C))) \neq (B\|((A\|C)\|(A\|C))) \ \mid\ \$Ans(A_SS)$ | [] |
| 2 | $((A\|A)\|(A\|B)) \neq A \ \mid\ \$Ans(B_SS)$ | [] |
| 3 | $(A\|(A\|A)) \neq (B\|(B\|B)) \ \mid\ \$Ans(ONE_SS)$ | [] |
| 77,76 | $((((x\|y)\|(y\|z))\|u)\|(y\|((y\|((x\|x)\|x))\|z))) = y$ | [] |
| 79,78 | $((x\|y)\|(x\|((x\|((((z\|x)\|(x\|u))\|((z\|x)\|(x\|u)))\|((z\|x)\|(x\|u))))\|((x\|((z\|z)\|z))\|u)))) = x$ | [76 → 76] |
| 80 | $(x\|((x\|y)\|(((x\|y)\|(((z\|x)\|(z\|x))\|(z\|x)))\|u))) = (x\|y)$ | [76 → 76] |
| 82 | $((x\|y)\|(((x\|y)\|(y\|z))\|y)) = ((x\|y)\|(y\|z))$ | [76 → 80] |
| 84 | $(x\|((x\|y)\|(((z\|x)\|(z\|x))\|(z\|x)))) = (x\|y)$ | [80 → 80] |
| 87,86 | $((((x\|y)\|(y\|z))\|y)\|(y\|y)) = y$ | [76 → 82,77] |
| 88 | $((x\|x)\|(x\|x)) = x$ | [86 → 86] |
| 90 | $(x\|((x\|((x\|x)\|y))\|(x\|x))) = (x\|((x\|x)\|y))$ | [86 → 82,87,87] |
| 92 | $(x\|((x\|y)\|(x\|(x\|x)))) = (x\|y)$ | [88 → 84] |
| 94 | $((((x\|y)\|(y\|(y\|(y\|y))))\|z)\|(y\|((x\|x)\|x))) = y$ | [92 → 76] |
| 96 | $(x\|((x\|y)\|(x\|y))) = (x\|y)$ | [78 → 80] |
| 99,98 | $((x\|y)\|(x\|x)) = x$ | [78 → 96,79,79] |
| 100 | $((((x\|y)\|(y\|z))\|u)\|(y\|y)) = y$ | [76 → 96,77,77] |
| 102 | $(x\|((x\|x)\|y)) = (x\|x)$ | [90:99] |
| 104 | $(((x\|x)\|y)\|x) = (x\|x)$ | [98 → 98] |
| 107,106 | $((x\|x)\|(x\|y)) = x$ | [98 → 102,99] |
| 108 | $\$Ans(B_SS)$ | [106,2] |
| 110,109 | $(((x\|y)\|(y\|z))\|((x\|y)\|(y\|z))) = (((x\|y)\|(y\|z))\|y)$ | [76 → 102] |
| 112 | $(((x\|y)\|(x\|y))\|x) = (x\|y)$ | [98 → 106] |
| 117,116 | $(((x\|y)\|(y\|z))\|y) = (y\|((x\|y)\|(y\|z)))$ | [76 → 104,110] |
| 119,118 | $(((x\|y)\|(y\|z))\|((x\|y)\|(y\|z))) = (y\|((x\|y)\|(y\|z)))$ | [109:117] |
| 122 | $((((x\|y)\|(x\|z))\|u)\|(x\|x)) = x$ | [112 → 100] |
| 124 | $(((x\|((x\|y)\|z))\|u)\|((x\|y)\|(x\|y))) = (x\|y)$ | [106 → 100] |

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|---|---|---|
| 127,126 | $((x\|y)\|(y\|y)) = y$ | [98 → 100] |
| 130 | $(x\|((y\|x)\|(y\|x))) = (y\|x)$ | [126 → 98] |
| 132 | $((x\|x)\|(y\|x)) = x$ | [126 → 112,127,127] |
| 135,134 | $(x\|(y\|(x\|x))) = (x\|x)$ | [132 → 132] |
| 137,136 | $((x\|y)\|(x\|((z\|z)\|z))) = x$ | [94:135,127] |
| 138 | $(((x\|((y\|x)\|z))\|u)\|((y\|x)\|(y\|x))) = (y\|x)$ | [132 → 100] |
| 140 | $((x\|((y\|z)\|(z\|u)))\|(z\|z)) = z$ | [130 → 100] |
| 142 | $((((x\|y)\|(z\|y))\|u)\|(y\|y)) = y$ | [130 → 100] |
| 145,144 | $(x\|((x\|y)\|((z\|z)\|z))) = (x\|y)$ | [136 → 136] |
| 146 | $(x\|x) = (x\|((y\|y)\|y))$ | [136 → 132,137] |
| 147 | $(x\|((y\|y)\|y)) = (x\|x)$ | [146] |
| 150 | $(x\|(y\|(y\|y))) = (x\|x)$ | [132 → 147] |
| 151 | $(x\|x) = (x\|(y\|(y\|y)))$ | [150] |
| 152 | $(x\|((x\|y)\|(z\|(z\|z)))) = (x\|y)$ | [151 → 96] |
| 154 | $((x\|(y\|(y\|y)))\|(z\|x)) = x$ | [151 → 132] |
| 156 | $((x\|((y\|z)\|(y\|u)))\|(y\|y)) = y$ | [130 → 122] |
| 159,158 | $((x\|(y\|((z\|y)\|u)))\|((z\|y)\|(z\|y))) = (z\|y)$ | [154 → 140] |
| 160 | $((((x\|y)\|(z\|y))\|u)\|(y\|((v\|v)\|v))) = y$ | [146 → 142] |
| 162 | $(((x\|(x\|x))\|(y\|(y\|y)))\|z) = (x\|(x\|x))$ | [154 → 152] |
| 164 | $(x\|(x\|x)) = (y\|(y\|y))$ | [126 → 162] |
| 165 | $\$Ans(ONE_SS)$ | [164,3] |
| 168 | $(((x\|y)\|z)\|((x\|((y\|u)\|(y\|v)))\|(x\|((y\|u)\|(y\|v))))) = (x\|((y\|u)\|(y\|v)))$ | [156 → 124] |
| 170 | $(((x\|y)\|z)\|((y\|x)\|(y\|x))) = (y\|x)$ | [136 → 138] |
| 172 | $((x\|y)\|(y\|x)) = ((y\|x)\|(y\|x))$ | [170 → 104] |
| 173 | $((x\|y)\|(((y\|x)\|z)\|((u\|u)\|u))) = ((y\|x)\|z)$ | [170 → 136] |
| 176,175 | $(x\|((y\|x)\|(x\|y))) = (y\|x)$ | [172 → 172,107,119] |
| 177 | $(x\|y) = (y\|x)$ | [172 → 172,99,119,176] |
| 178 | $(((x\|y)\|z)\|((y\|x)\|(x\|y))) = (y\|x)$ | [177 → 170] |
| 180 | $(((x\|y)\|z)\|((x\|y)\|(y\|x))) = (y\|x)$ | [177 → 170] |
| 186 | $((x\|(y\|z))\|((((z\|u)\|(z\|v))\|y)\|(((z\|u)\|(z\|v))\|y))) = (((z\|u)\|(z\|v))\|y)$ | [122 → 158] |
| 188 | $((x\|y)\|z) = ((y\|x)\|z)$ | [177 → 173,145] |
| 189 | $(x\|(y\|z)) = ((z\|y)\|x)$ | [177 → 188] |
| 190 | $((x\|y)\|z) = (z\|(y\|x))$ | [189] |
| 191 | $(x\|((y\|z)\|u)) = (((z\|y)\|u)\|x)$ | [189 → 189] |
| 192 | $(((x\|y)\|z)\|u) = (u\|((y\|x)\|z))$ | [191] |
| 194,193 | $((((x\|y)\|z)\|u)\|(((y\|x)\|z)\|((x\|y)\|z))) = (z\|(x\|y))$ | [189 → 178] |
| 195 | $((x\|y)\|(z\|u)) = ((y\|x)\|(u\|z))$ | [189 → 190] |
| 196 | $((((x\|y)\|(z\|u))\|v)\|(((y\|x)\|(u\|z))\|((z\|u)\|(x\|y)))) = ((z\|u)\|(x\|y))$ | [195 → 180] |
| 198 | $((((x\|(y\|((z\|y)\|u)))\|z)\|v)\|((z\|y)\|(z\|y))) = (z\|y)$ | [158 → 168,159,159] |
| 200 | $((x\|(((y\|((z\|y)\|u))\|v)\|z))\|((z\|y)\|(z\|y))) = (z\|y)$ | [192 → 198] |
| 202 | $((x\|(y\|z))\|((((z\|u)\|(z\|v))\|y)\|(((z\|v)\|(z\|u))\|y))) = (((z\|v)\|(z\|u))\|y)$ | [177 → 186] |
| 204 | $(((((x\|y)\|(x\|z))\|u)\|(((x\|z)\|(x\|y))\|u))\|((u\|x)\|v)) = (((x\|z)\|(x\|y))\|u)$ | [190 → 202] |
| 206 | $(((x\|((y\|x)\|z))\|(x\|u))\|y) = (y\|x)$ | [200 → 204] |
| 208 | $(((x\|y)\|(x\|z))\|((u\|y)\|(v\|y))) = (((u\|y)\|(v\|y))\|x)$ | [160 → 206] |
| 210,209 | $(((x\|y)\|(z\|y))\|((u\|v)\|(u\|y))) = (((x\|y)\|(z\|y))\|u)$ | [190 → 208] |
| 212,211 | $(((x\|y)\|(z\|y))\|((u\|y)\|(u\|v))) = (((x\|y)\|(z\|y))\|u)$ | [177 → 208] |
| 217 | $(((x\|y)\|(x\|z))\|((u\|y)\|(v\|y))) = (x\|((u\|y)\|(v\|y)))$ | [208 → 196,212,210,194] |
| 229 | $(((x\|y)\|(x\|y))\|z) = (x\|((z\|y)\|(z\|y)))$ | [211 → 217] |
| 230 | $(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$ | [190 → 229] |
| 231 | $\$Ans(A_SS)$ | [230,1] |

**Proof OL-Sh-sound**

| | | |
|---|---|---|
| 2 | $(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$ | [] |
| 4,3 | $((x\|x)\|(x\|y)) = x$ | [] |
| 6 | $(x\|(x\|x)) = 1$ | [] |
| 8 | $((((B\|A)\|(A\|C))\|D)\|(A\|((A\|((B\|B)\|B))\|C))) \neq A \mid \$Ans(OL_Sh)$ | [] |
| 13 | $(x\|((x\|x)\|y)) = (x\|x)$ | [3 → 3] |
| 15 | $((x\|x), 1) = x$ | [6 → 3] |
| 20,19 | $(x\|((((x\|y)\|(x\|y))\|y)\|(((x\|y)\|(x\|y))\|y))) = (x\|y)$ | [2 → 3] |
| 22,21 | $((x\|x)\|x) = 1$ | [3 → 6] |
| 23 | $(x\|((y\|x)\|(y\|x))) = (y\|x)$ | [3 → 2] |
| 25 | $((x\|x)\|((y\|(x\|z))\|(y\|(x\|z)))) = (y\|(x\|x))$ | [3 → 2,4] |
| 27 | $((((B\|A)\|(A\|C))\|D)\|(A\|((A,1)\|C))) \neq A \mid \$Ans(OL_Sh)$ | [8:22] |
| 28 | $(x,1) = (x\|x)$ | [3 → 15] |
| 30,29 | $((x\|y)\|(x\|y)) = ((x\|y),1)$ | [2 → 15,20] |
| 31 | $(x\|x) = (x,1)$ | [28] |
| 32 | $((x\|x)\|((y\|(x\|z)),1)) = (y\|(x\|x))$ | [25:30] |
| 34 | $(x\|((y\|x),1)) = (y\|x)$ | [23:30] |
| 44 | $(x\|((y\|z),1)) = (y\|((x\|z),1))$ | [2:30,30] |
| 45 | $((x,1)\|x) = 1$ | [31 → 21] |
| 48,47 | $((x,1),1) = x$ | [31 → 15] |
| 50,49 | $(x\|((x,1)\|y)) = (x\|x)$ | [31 → 13] |
| 55 | $((((B\|A)\|(A\|C))\|D)\|(A\|A)) \neq A \mid \$Ans(OL_Sh)$ | [27:50] |
| 60 | $(x\|y) = (y\|x)$ | [45 → 32,30,48,48,30,48] |
| 68,67 | $((x\|y)\|(x\|x)) = x$ | [31 → 32,48,4] |
| 76,75 | $(1\|(x,1)) = x$ | [47 → 60] |
| 79 | $(x\|x) = (1\|x)$ | [28 → 60] |
| 88,87 | $((1\|x),1) = x$ | [60 → 47] |
| 91 | $((x\|x)\|(1\|(y\|(x\|z)))) = (y\|(x\|x))$ | [60 → 32] |
| 95,94 | $((x\|x)\|(y\|x)) = x$ | [60 → 3] |
| 96 | $((x\|x)\|((y\|(z\|x)),1)) = (y\|(x\|x))$ | [60 → 32] |
| 100,99 | $(1\|(1\|x)) = x$ | [60 → 75] |
| 113 | $(x\|((x\|y),1)) = (y\|x)$ | [60 → 34] |
| 115 | $((x\|y)\|(x,1)) = x$ | [3 → 34,4] |
| 166 | $((x\|y)\|(y\|y)) = y$ | [60 → 67] |
| 178 | $(((x\|y),1)\|((z\|x),1)) = (z\|((x\|y),1))$ | [67 → 32,30,30] |
| 180 | $((1\|x)\|(y\|x)) = x$ | [79 → 94] |
| 184 | $((x\|(y\|z))\|(x\|(y\|y))) = ((x\|(y\|z)),1)$ | [32 → 94,30,48] |
| 213 | $(x\|(y,1)) = ((y\|y)\|x)$ | [15 → 44,48] |
| 221 | $((x\|x)\|y) = (y\|(x,1))$ | [213] |
| 267 | $((x\|(y,1))\|y) = (y,1)$ | [115 → 166] |
| 269 | $((x\|y)\|(1\|y)) = y$ | [79 → 166] |
| 340 | $((((A\|B)\|(A\|C))\|D)\|(A\|A)) \neq A \mid \$Ans(OL_Sh)$ | [60 → 55] |
| 391 | $((x\|y)\|(y\|x)) = ((x\|y),1)$ | [113 → 180,76] |
| 810,809 | $((x\|(y\|y))\|(x\|(y\|z))) = (1\|(x\|(y\|z)))$ | [91 → 269,100] |
| 1000 | $((x\|((y\|z),1))\|(z\|z)) = z$ | [267 → 96,48,95] |
| 2382 | $((x\|((y\|z),1))\|(x\|y)) = ((x\|y),1)$ | [178 → 267] |
| 2840,2839 | $((x\|(y\|y)),1) = (1\|(x\|(y,1)))$ | [221 → 391,810] |
| 3192 | $((x\|(y\|(z\|u)))\|(1\|(y\|(z,1)))) = (y\|(z\|z))$ | [184 → 1000,48,30,2840] |
| 4965 | $((D\|((A\|B)\|(A\|C)))\|(A\|A)) \neq A \mid \$Ans(OL_Sh)$ | [60 → 340] |
| 20958 | $((D\|((A\|B)\|(A\|C)))\|(1\|A)) \neq A \mid \$Ans(OL_Sh)$ | [79 → 4965] |
| 38024 | $((x\|((y\|z)\|(y\|u)))\|(1\|y)) = y$ | [2382 → 3192,88,48,88,68] |
| 38026 | $\$Ans(OL_Sh)$ | [38024,20958] |

### 3.5.2 A Single Axiom for Orthomodular Lattices

As in the $\mathcal{OL}$ case, we list three Otter proofs:

1. OML-Sh implies the join/complement basis {AJ, B1, DM, OM}.
2. OML-Sh implies the Sheffer stroke basis {A_SS, B_SS, OM_SS}.
3. The Sheffer stroke basis {A_SS, B_SS, OM_SS} implies OML-Sh.

The first two prove essentially the same theorem—either one is sufficient. For the first proof we have rewritten goals B1 and OM to eliminate the meet operation; this allows a convenient forward proof.

**Proof OML-Sh-jc**

| | | |
|---|---|---|
| 1 | $A \vee (B \vee C) \neq B \vee (A \vee C) \mid \$Ans(AJ)$ | [] |
| 2 | $A \vee c(c(A) \vee B) \neq A \mid \$Ans(B1_rewritten)$ | [] |
| 3 | $A \wedge B \neq c(c(A) \vee c(B)) \mid \$Ans(DM)$ | [] |
| 4 | $A \vee c(A \vee c(A \vee B)) \neq A \vee B \mid \$Ans(OM_rewritten)$ | [] |
| 5 | $(A\|B) \neq c(A) \vee c(B) \mid \$Ans(DEF_SS)$ | [] |
| 7 | $((((x\|y)\|(y\|z))\|u)\|(y\|((z\|((y\|y)\|z))\|z))) = y$ | [] |
| 9 | $x \vee y = ((x\|x)\|(y\|y))$ | [] |
| 10 | $((x\|x)\|(y\|y)) = x \vee y$ | [9] |
| 12 | $x \wedge y = ((x\|y)\|(x\|y))$ | [] |
| 13 | $((x\|y)\|(x\|y)) = x \wedge y$ | [12] |
| 15 | $c(x) = (x\|x)$ | [] |
| 17,16 | $(x\|x) = c(x)$ | [15] |
| 19,18 | $c((x\|y)) = x \wedge y$ | [13:17] |
| 20 | $(c(x)\|c(y)) = x \vee y$ | [10:17,17] |
| 22 | $((((x\|y)\|(y\|z))\|u)\|(y\|((z\|(c(y)\|z))\|z))) = y$ | [7:17] |
| 30,29 | $c(c(x)) = x \vee x$ | [16 → 20] |
| 34 | $c(x) \wedge c(y) = c(x \vee y)$ | [20 → 18] |
| 37,36 | $c(x \vee x) = c(x) \vee c(x)$ | [29 → 29] |
| 40 | $(c(x)\|y \vee y) = x \vee c(y)$ | [29 → 20] |
| 43,42 | $(x \vee x\|c(y)) = c(x) \vee y$ | [29 → 20] |
| 46 | $(((c(x)\|(x\|y))\|z)\|(x\|((y\|(c(x)\|y))\|y))) = x$ | [16 → 22] |
| 48 | $((((x\|c(y))\|y \vee z)\|u)\|(c(y)\|((c(z)\|c(y) \vee z)\|c(z)))) = c(y)$ | [20 → 22,30,43] |
| 50 | $((((x\|y)\|c(y))\|z)\|(y\|((y\|(c(y)\|y))\|y))) = y$ | [16 → 22] |
| 52 | $((x\|y)\|(x\|((((z\|(c(x)\|z))\|z)\|(c(x)\|((z\|(c(x)\|z))\|z)))\|((z\|(c(x)\|z))\|z)))) = x$ | [22 → 22] |
| 54 | $((x \vee x\|y)\|(x\|((x\|(c(x)\|x))\|x))) = x$ | [16 → 22,17,30] |
| 70 | $(x \vee x) \wedge c(y) = c(c(x) \vee y)$ | [29 → 34] |
| 83,82 | $(x \vee x\|y \vee y) = c(x) \vee c(y)$ | [29 → 40] |
| 90 | $(((x \vee c(y)\|(y \vee y\|z))\|u)\|(y \vee y\|((z\|(c(y) \vee c(y)\|z))\|z))) = y \vee y$ | [40 → 22,37] |
| 136 | $(x \vee x) \wedge (y \vee y) = c(c(x) \vee c(y))$ | [29 → 70] |
| 158 | $(((c(x) \vee c(x)\|c(x) \vee y)\|z)\|(x \vee x\|((c(y)\|(x \vee x) \vee y)\|c(y)))) = x \vee x$ | [42 → 46,37,37,43,30] |
| 234 | $((x\|c(y)) \wedge (y \vee z)\|(c(y)\|((c(z)\|c(y) \vee z)\|c(z)))) = c(y)$ | [16 → 48,19] |
| 306 | $(((x \vee c(y)\|c(y) \vee c(y))\|z)\|(y \vee y\|((y \vee y\|(y \vee y) \vee c(y))\|y \vee y))) = y \vee y$ | [40 → 50,37,37,83,30] |
| 398 | $(c(x)\|(x\|((y\|(c(x)\|y))\|y))) = x$ | [52 → 46] |
| 403,402 | $((x\|y)\|(y\|((z\|(c(y)\|z))\|z))) = y$ | [52 → 22] |
| 451,450 | $x \vee x = x$ | [398 → 398,403,17,17,30] |
| 456 | $(c(x)\|(x\|((c(y)\|x \vee y)\|c(y)))) = x$ | [20 → 398] |
| 536 | $(((x \vee c(y)\|c(y))\|z)\|(y\|((y\|y \vee c(y))\|y))) = y$ | [306:451,451,451,451,451,451] |

| | | |
|---|---|---|
| 600 | $(((c(x)\|c(x) \vee y)\|z)\|(x\|((c(y)\|x \vee y)\|c(y)))) = x$ | [158:451,451,451,451] |
| 603,602 | $x \wedge y = c(c(x) \vee c(y))$ | [136:451,451] |
| 604 | $\$Ans(DM)$ | [602,3] |
| 623 | $(((x \vee c(y)\|(y\|z))\|u)\|(y\|((z\|(c(y)\|z))\|z))) = y$ | [90:451,451,451,451] |
| 632,631 | $(x\|y) = c(x) \vee c(y)$ | [82:451,451] |
| 633 | $\$Ans(DEF_SS)$ | [631,5] |
| 638 | $c(c(x) \vee c(y)) \vee c(c(x) \vee c(c(c(x) \vee c(x \vee c(x))) \vee c(x))) = x$ | [54:451,632,632,30,451,632,632,632,632] |
| 643,642 | $c(c(x)) = x$ | [29:451] |
| 654 | $(c(c(x) \vee y) \vee c(y \vee z)) \vee c(y \vee c(c(z \vee c(c(y) \vee z)) \vee z)) = c(y)$ | [234:632,643,603,632,643,632,643,632,643,632,643] |
| 666 | $c(c(c(x \vee c(y)) \vee c(c(y) \vee c(z))) \vee c(u)) \vee c(c(y) \vee c(c(c(z) \vee c(y \vee c(z))) \vee c(z))) = y$ | [623:632,632,632,632,643,632,632,632,632] |
| 672 | $c(c(x \vee c(c(x) \vee y)) \vee c(z)) \vee c(c(x) \vee c(c(y \vee c(x \vee y)) \vee y)) = x$ | [600:632,643,632,632,643,632,643,632,632] |
| 674 | $c(c(c(x \vee c(y)) \vee y) \vee c(z)) \vee c(c(y) \vee c(c(c(y) \vee c(y \vee c(y))) \vee c(y))) = y$ | [536:632,643,632,632,632,632,632] |
| 682 | $x \vee c(c(x) \vee c(c(y \vee c(x \vee y)) \vee y)) = x$ | [456:632,643,632,643,632,632,643] |
| 706 | $c(c(x) \vee y) \vee c(c(x) \vee c(c(c(x) \vee c(x \vee c(x))) \vee c(x))) = x$ | [642 → 638] |
| 709,708 | $c(c(x) \vee c(c(c(x) \vee c(x \vee c(x))) \vee c(x))) = x$ | [450 → 638] |
| 712 | $c(c(x) \vee y) \vee x = x$ | [706:709] |
| 714 | $c(c(c(x \vee c(y)) \vee y) \vee c(z)) \vee y = y$ | [674:709] |
| 719,718 | $c(x \vee y) \vee c(x) = c(x)$ | [642 → 712] |
| 724 | $x \vee (x \vee y) = x \vee y$ | [718 → 718,643,643,643] |
| 728 | $x \vee c(c(x) \vee c(c((x \vee y) \vee c(x \vee y)) \vee (x \vee y))) = x$ | [724 → 682] |
| 738 | $(x \vee c(c(x) \vee y)) \vee c(c(x) \vee c(c(y \vee c(x \vee y)) \vee y)) = x$ | [718 → 654,643,643,643] |
| 794 | $c((c(x) \vee c(y)) \vee c(z)) \vee c(c(x) \vee c(c(c(y) \vee c(x \vee c(y))) \vee c(y))) = x$ | [718 → 666,643] |
| 800 | $(c(x \vee c(y)) \vee c(c(y) \vee c(z))) \vee c(c(y) \vee c(c(c(z) \vee c(y \vee c(z))) \vee c(z))) = y$ | [682 → 666,643] |
| 809,808 | $c(x \vee c(y)) \vee y = y$ | [718 → 714,643] |
| 818 | $x \vee c(c(x) \vee c(x \vee y)) = x$ | [728:809] |
| 822 | $c(x \vee y) \vee c(y) = c(y)$ | [642 → 808] |
| 839,838 | $x \vee (y \vee x) = y \vee x$ | [822 → 712,643] |
| 877,876 | $c(c(x) \vee c(c(y \vee c(x \vee y)) \vee y)) = x$ | [808 → 672] |
| 892 | $(c(x \vee c(y)) \vee c(c(y) \vee c(z))) \vee y = y$ | [800:877] |
| 896 | $c((c(x) \vee c(y)) \vee c(z)) \vee x = x$ | [794:877] |
| 909,908 | $(x \vee c(c(x) \vee y)) \vee x = x$ | [738:877] |
| 913,912 | $(x \vee c(y \vee c(x))) \vee x = x$ | [838 → 908] |
| 919,918 | $c(x) \vee c(x \vee c(c(x) \vee y)) = c(x \vee c(c(x) \vee y))$ | [908 → 718] |
| 925,924 | $c(x \vee c(c(x) \vee y)) = c(x)$ | [642 → 818,919] |
| 926 | $x \vee c(y \vee c(x)) = x$ | [912 → 818,719,643,913] |
| 928 | $x \vee c(c(x) \vee y) = x$ | [908 → 818,925,451,643,909] |
| 930 | $\$Ans(B1_rewritten)$ | [928,2] |
| 937 | $c(x) \vee c(y \vee x) = c(x)$ | [642 → 926] |
| 953 | $c(x) \vee c(x \vee y) = c(x)$ | [642 → 928] |
| 955 | $(x \vee y) \vee y = x \vee y$ | [822 → 928,643] |
| 958,957 | $(x \vee y) \vee x = x \vee y$ | [718 → 928,643] |
| 1017 | $c((c(x) \vee y) \vee c(z)) \vee x = x$ | [642 → 896] |
| 1035 | $c((c(x) \vee y) \vee z) \vee x = x$ | [642 → 1017] |
| 1037 | $c((x \vee y) \vee z) \vee c(x) = c(x)$ | [642 → 1035] |
| 1041 | $c((x \vee c(y)) \vee z) \vee y = y$ | [838 → 1035] |
| 1051 | $c(x \vee (c(y) \vee z)) \vee y = y$ | [838 → 1035] |
| 1063 | $c(c(x \vee y) \vee c(c(y \vee c(x \vee y)) \vee y)) = x \vee y$ | [955 → 876] |
| 1085 | $c(x \vee (y \vee c(z))) \vee z = z$ | [838 → 1041] |
| 1095 | $c(x \vee (y \vee z)) \vee c(y) = c(y)$ | [642 → 1051] |

| | | |
|---|---|---|
| 1159 | $(c(x \vee c(y \vee (z \vee u))) \vee u) \vee (y \vee (z \vee u)) = y \vee (z \vee u)$ | [1085 → 892,643,643,643,643] |
| 1161 | $(c(x \vee c(y \vee (z \vee u))) \vee z) \vee (y \vee (z \vee u)) = y \vee (z \vee u)$ | [1051 → 892,643,643,643,643] |
| 1163 | $(c(x \vee c((y \vee z) \vee u)) \vee z) \vee ((y \vee z) \vee u) = (y \vee z) \vee u$ | [1041 → 892,643,643,643,643] |
| 1165 | $(c(x \vee c((y \vee z) \vee u)) \vee y) \vee ((y \vee z) \vee u) = (y \vee z) \vee u$ | [1035 → 892,643,643,643,643] |
| 1168,1167 | $(c(x \vee c(y \vee z)) \vee z) \vee (y \vee z) = y \vee z$ | [822 → 892,643] |
| 1169 | $(c(x \vee c(y \vee z)) \vee y) \vee (y \vee z) = y \vee z$ | [718 → 892,643] |
| 1200,1199 | $((x \vee y) \vee z) \vee x = (x \vee y) \vee z$ | [1037 → 928,643] |
| 1258,1257 | $x \vee (y \vee (x \vee z)) = y \vee (x \vee z)$ | [1095 → 712,643] |
| 2405 | $c(x \vee c(y \vee x)) \vee x = y \vee x$ | [1063 → 937,643,1168,643] |
| 2511 | $c(x \vee c(x \vee y)) \vee x = x \vee y$ | [957 → 2405,958] |
| 4760 | $(x \vee y) \vee (x \vee (z \vee y)) = x \vee (z \vee y)$ | [953 → 1159,643] |
| 5313,5312 | $(x \vee y) \vee (x \vee (y \vee z)) = x \vee (y \vee z)$ | [953 → 1161,643] |
| 5739,5738 | $(x \vee y) \vee ((z \vee y) \vee x) = (z \vee y) \vee x$ | [937 → 1163,643] |
| 6102 | $(x \vee y) \vee ((y \vee z) \vee x) = (y \vee z) \vee x$ | [937 → 1165,643] |
| 7373,7372 | $(x \vee y) \vee (y \vee x) = y \vee x$ | [937 → 1169,643] |
| 7594 | $x \vee y = y \vee x$ | [7372 → 957,7373,7373] |
| 7609 | $x \vee c(x \vee c(x \vee y)) = x \vee y$ | [2511 → 7594] |
| 7611 | $\$Ans(OM_rewritten)$ | [7609,4] |
| 17813 | $(x \vee (y \vee z)) \vee (x \vee z) = x \vee (y \vee z)$ | [7594 → 4760] |
| 18921 | $((x \vee y) \vee (y \vee z)) \vee (x \vee (y \vee z)) = x \vee (y \vee z)$ | [5312 → 4760,5313] |
| 19351 | $((x \vee y) \vee (z \vee y)) \vee ((z \vee y) \vee x) = (z \vee y) \vee x$ | [5738 → 5312,5739] |
| 19774,19773 | $((x \vee y) \vee z) \vee (z \vee x) = (x \vee y) \vee z$ | [7594 → 6102] |
| 19779 | $(x \vee y) \vee (z \vee y) = (z \vee y) \vee x$ | [19351:19774] |
| 20260 | $(x \vee y) \vee (z \vee x) = (z \vee x) \vee y$ | [7594 → 19779] |
| 20410,20409 | $(x \vee y) \vee (z \vee y) = (x \vee y) \vee z$ | [7594 → 19779] |
| 20422 | $(x \vee y) \vee z = (y \vee z) \vee (x \vee y)$ | [20260] |
| 20472,20471 | $(x \vee y) \vee (y \vee z) = x \vee (y \vee z)$ | [18921:20410,1200] |
| 20614,20613 | $(x \vee y) \vee (z \vee x) = (x \vee y) \vee z$ | [19773:20472] |
| 20648 | $(x \vee y) \vee z = (y \vee z) \vee x$ | [20422:20614] |
| 21300,21299 | $(x \vee y) \vee z = x \vee (y \vee z)$ | [20648 → 724,20614,20472] |
| 22049 | $x \vee (y \vee z) = y \vee (x \vee z)$ | [17813:21300,21300,839,1258] |
| 22050 | $\$Ans(AJ)$ | [22049,1] |

**Proof OML-Sh**

| | | |
|---|---|---|
| 1 | $(A\|((B\|C)\|(B\|C))) \neq (B\|((A\|C)\|(A\|C))) \ \mid \ \$Ans(A_SS)$ | [] |
| 2 | $((A\|A)\|(A\|B)) \neq A \ \mid \ \$Ans(B_SS)$ | [] |
| 3 | $(A\|(A\|(A\|B))) \neq (A\|B) \ \mid \ \$Ans(OM_SS)$ | [] |
| 77,76 | $((((x\|y)\|(y\|z))\|u)\|(y\|((z\|((y\|y)\|z))\|z))) = y$ | [] |
| 78 | $(x\|((x\|y)\|((z\|(((x\|y)\|(x\|y))\|z))\|z))) = (x\|y)$ | [76 → 76] |
| 80 | $((((x\|y)\|(y\|z))\|u)\|(y\|((v\|((y\|y)\|v))\|v))) = y$ | [76 → 78,77,77,77] |
| 83,82 | $((x\|y)\|(x\|((z\|((x\|x)\|z))\|z))) = x$ | [80 → 80] |
| 85,84 | $((x\|y)\|(y\|((z\|((y\|y)\|z))\|z))) = y$ | [82 → 80] |
| 87,86 | $((x\|y)\|(y\|y)) = y$ | [84 → 84,85] |
| 88 | $((x\|y)\|(y\|((y\|y)\|z))) = y$ | [78 → 84,85] |
| 91,90 | $((x\|y)\|(x\|x)) = x$ | [84 → 82,85] |
| 92 | $((((x\|y)\|(y\|z))\|u)\|(y\|y)) = y$ | [84 → 80,85] |
| 94 | $(x\|((x\|y)\|(x\|y))) = (x\|y)$ | [84 → 78,85] |
| 96 | $(x\|((y\|x)\|((z\|(((y\|x)\|(y\|x))\|z))\|z))) = (y\|x)$ | [84 → 82] |
| 99,98 | $(x\|((y\|((x\|x)\|y))\|y)) = (x\|x)$ | [84 → 86,83] |
| 100 | $(x\|((y\|x)\|(y\|x))) = (y\|x)$ | [96:99] |
| 102 | $(((x\|x)\|y)\|x) = (x\|x)$ | [90 → 90] |

| | | |
|---|---|---|
| 105,104 | $((x\|((x\|x)\|y))\|(x\|((x\|x)\|y))) = x$ | [102 → 88] |
| 106 | $(x\|((x\|x)\|y)) = (x\|x)$ | [88 → 86,105] |
| 109,108 | $((x\|x)\|(x\|y)) = x$ | [90 → 106,91] |
| 110 | $\$Ans(B_SS)$ | [108,2] |
| 112,111 | $(((x\|y)\|(x\|y))\|x) = (x\|y)$ | [90 → 108] |
| 113 | $((x\|x)\|(y\|x)) = x$ | [100 → 108] |
| 115 | $(((x\|((y\|x)\|z))\|u)\|((y\|x)\|(y\|x))) = (y\|x)$ | [113 → 92] |
| 117 | $((((x\|y)\|(x\|z))\|u)\|(x\|x)) = x$ | [111 → 92] |
| 119 | $(((x\|((x\|y)\|z))\|u)\|((x\|y)\|(x\|y))) = (x\|y)$ | [108 → 92] |
| 121 | $((((x\|y)\|(z\|y))\|u)\|(y\|y)) = y$ | [100 → 92] |
| 123 | $((x\|((y\|z)\|(z\|u)))\|(z\|z)) = z$ | [100 → 92] |
| 126,125 | $(((x\|y)\|(y\|z))\|((x\|y)\|(y\|z))) = (y\|((x\|y)\|(y\|z)))$ | [92 → 102] |
| 127 | $(x\|(((y\|x)\|x)\|(y\|x))) = (x\|x)$ | [113 → 98] |
| 129 | $((x\|((y\|z)\|(y\|u)))\|(y\|y)) = y$ | [100 → 117] |
| 132,131 | $((x\|((y\|z)\|(u\|z)))\|(z\|z)) = z$ | [100 → 121] |
| 134,133 | $((x\|(y\|((z\|y)\|u)))\|((z\|y)\|(z\|y))) = (z\|y)$ | [113 → 123] |
| 135 | $(((x\|y)\|y)\|(x\|y)) = y$ | [127 → 113,132] |
| 137 | $(((x\|y)\|x)\|(x\|y)) = x$ | [111 → 135,112] |
| 139 | $((x\|(x\|y))\|x) = (x\|y)$ | [108 → 135,109] |
| 141 | $((x\|y)\|(x\|(x\|y))) = x$ | [139 → 137] |
| 143 | $(((x\|y)\|z)\|((y\|x)\|(y\|x))) = (y\|x)$ | [141 → 115] |
| 145 | $((x\|y)\|(y\|x)) = ((y\|x)\|(y\|x))$ | [143 → 102] |
| 147,146 | $(x\|((y\|x)\|(x\|y))) = (y\|x)$ | [145 → 145,109,126] |
| 148 | $(x\|y) = (y\|x)$ | [145 → 145,91,126,147] |
| 150,149 | $((((x\|y)\|(y\|x))\|z)\|(x\|y)) = ((y\|x)\|(x\|y))$ | [145 → 143,87] |
| 151 | $(x\|(x\|(x\|y))) = (x\|y)$ | [139 → 148] |
| 153 | $\$Ans(OM_SS)$ | [151,3] |
| 154 | $((x\|(y\|x))\|(y\|x)) = x$ | [148 → 135] |
| 157,156 | $(((x\|y)\|z)\|((y\|x)\|(x\|y))) = (y\|x)$ | [148 → 143] |
| 158 | $(((x\|y)\|z)\|((x\|y)\|(y\|x))) = (y\|x)$ | [148 → 143] |
| 161,160 | $((x\|(x\|y))\|(y\|x)) = x$ | [148 → 154] |
| 163,162 | $(((x\|y)\|(y\|x))\|((y\|x)\|z)) = (y\|x)$ | [156 → 156,157,150,157] |
| 164 | $(((x\|y)\|z)\|((x\|((y\|u)\|(y\|v)))\|(x\|((y\|u)\|(y\|v))))) = (x\|((y\|u)\|(y\|v)))$ | [129 → 119] |
| 166 | $((x\|(y\|z))\|((((z\|u)\|(z\|v))\|y)\|(((z\|u)\|(z\|v))\|y))) = (((z\|u)\|(z\|v))\|y)$ | [117 → 133] |
| 168 | $((x\|y)\|(y\|((x\|y)\|(y\|z)))) = ((x\|y)\|(y\|z))$ | [125 → 94] |
| 171,170 | $((x\|y)\|(y\|((x\|y)\|(z\|y)))) = ((x\|y)\|(y\|z))$ | [148 → 168] |
| 172 | $((x\|y)\|(y\|z)) = ((x\|y)\|(z\|y))$ | [146 → 168,171,147] |
| 173 | $(x\|((y\|x)\|z)) = (x\|(z\|(y\|x)))$ | [160 → 172,161] |
| 175 | $(x\|(y\|z)) = (x\|(z\|y))$ | [156 → 173,163] |
| 176 | $((x\|y)\|z) = (z\|(y\|x))$ | [148 → 175] |
| 177 | $(x\|(y\|z)) = ((z\|y)\|x)$ | [176] |
| 178 | $(x\|((y\|z)\|u)) = (((z\|y)\|u)\|x)$ | [177 → 177] |
| 179 | $((x\|y)\|(z\|u)) = ((y\|x)\|(u\|z))$ | [176 → 177] |
| 181 | $(((x\|y)\|z)\|u) = (u\|((y\|x)\|z))$ | [178] |
| 183,182 | $((((x\|y)\|z)\|u)\|(((y\|x)\|z)\|((x\|y)\|z))) = (z\|(x\|y))$ | [177 → 156] |
| 184 | $((((x\|y)\|(z\|u))\|v)\|(((y\|x)\|(u\|z))\|((z\|u)\|(x\|y)))) = ((z\|u)\|(x\|y))$ | [179 → 158] |
| 186 | $((((x\|(y\|((z\|y)\|u)))\|z)\|v)\|((z\|y)\|(z\|y))) = (z\|y)$ | [133 → 164,134,134] |
| 188 | $((x\|(((y\|((z\|y)\|u))\|v)\|z))\|((z\|y)\|(z\|y))) = (z\|y)$ | [181 → 186] |
| 190 | $((x\|(y\|z))\|((((z\|u)\|(z\|v))\|y)\|(((z\|v)\|(z\|u))\|y))) = (((z\|v)\|(z\|u))\|y)$ | [148 → 166] |
| 192 | $(((((x\|y)\|(x\|z))\|u)\|(((x\|z)\|(x\|y))\|u))\|((u\|x)\|v)) = (((x\|z)\|(x\|y))\|u)$ | [176 → 190] |
| 194 | $(((x\|((y\|x)\|z))\|(x\|u))\|y) = (y\|x)$ | [188 → 192] |
| 196 | $(((x\|y)\|(x\|z))\|((u\|y)\|(v\|y))) = (((u\|y)\|(v\|y))\|x)$ | [121 → 194] |
| 198,197 | $(((x\|y)\|(z\|y))\|((u\|v)\|(u\|y))) = (((x\|y)\|(z\|y))\|u)$ | [176 → 196] |
| 200,199 | $(((x\|y)\|(z\|y))\|((u\|y)\|(u\|v))) = (((x\|y)\|(z\|y))\|u)$ | [148 → 196] |

| | | |
|---|---|---|
| 205 | $(((x\|y)\|(x\|z))\|((u\|y)\|(v\|y))) = (x\|((u\|y)\|(v\|y)))$ | [196 → 184,200,198,183] |
| 215 | $(((x\|y)\|(x\|y))\|z) = (x\|((z\|y)\|(z\|y)))$ | [199 → 205] |
| 216 | $(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$ | [176 → 215] |
| 217 | $\$Ans(A_SS)$ | [216,1] |

**Proof OML-Sh-sound**

| | | |
|---|---|---|
| 2 | $(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$ | [] |
| 4,3 | $((x\|x)\|(x\|y)) = x$ | [] |
| 5 | $(x\|(x\|(x\|y))) = (x\|y)$ | [] |
| 7 | $((((B\|A)\|(A\|C))\|D)\|(A\|((C\|((A\|A)\|C))\|C))) \neq A \mid \$Ans(OML_Sh)$ | [] |
| 9,8 | $(x\|((x\|x)\|y)) = (x\|x)$ | [3 → 3] |
| 13,12 | $(x\|((y\|x)\|(y\|x))) = (y\|x)$ | [3 → 2] |
| 14 | $((x\|x)\|((y\|(x\|z))\|(y\|(x\|z)))) = (y\|(x\|x))$ | [3 → 2,4] |
| 31,30 | $((x\|y)\|(x\|x)) = x$ | [3 → 12,4,4] |
| 33,32 | $(x\|(y\|(x\|x))) = (x\|x)$ | [12 → 8] |
| 34 | $(x\|(x\|(y\|x))) = (y\|x)$ | [12 → 5,13] |
| 45,44 | $(x\|((((x\|y)\|z)\|y)\|(((x\|y)\|z)\|y))) = (x\|y)$ | [2 → 30] |
| 46 | $((x\|y)\|((x\|y)\|x)) = x$ | [30 → 5,31] |
| 48 | $(((x\|y)\|(x\|y))\|x) = (x\|y)$ | [30 → 3] |
| 50 | $((x\|y)\|((z\|(x\|x))\|(z\|(x\|x)))) = (z\|(x\|x))$ | [30 → 2,31] |
| 99 | $((x\|x)\|y) = (y\|(x\|x))$ | [32 → 14,33,4] |
| 103 | $((x\|(y\|z))\|(x\|(y\|y))) = ((x\|(y\|z))\|(x\|(y\|z)))$ | [14 → 32] |
| 117 | $(x\|y) = (y\|x)$ | [99 → 99,4,4] |
| 127,126 | $(((x\|y)\|x)\|(x\|y)) = x$ | [46 → 117] |
| 129,128 | $((x\|(y\|x))\|x) = (y\|x)$ | [34 → 117] |
| 133 | $((((B\|A)\|(A\|C))\|D)\|(A\|A)) \neq A \mid \$Ans(OML_Sh)$ | [7:129,9] |
| 146,145 | $(x\|((y\|x)\|(x\|y))) = (y\|x)$ | [117 → 12] |
| 197 | $(((x\|y)\|x)\|x) = (x\|y)$ | [126 → 46] |
| 199 | $(((x\|y)\|x)\|((z\|(x\|y))\|(z\|(x\|y)))) = (z\|(x\|x))$ | [126 → 2,127] |
| 338,337 | $(((x\|y)\|(y\|x))\|y) = (y\|x)$ | [117 → 48] |
| 342,341 | $(((x\|y)\|(y\|x))\|((x\|y)\|(y\|x))) = (x\|y)$ | [145 → 145,338,146] |
| 366,365 | $((x\|y)\|((z\|(y\|x))\|(z\|(y\|x)))) = (z\|(x\|y))$ | [337 → 14,342,338,342] |
| 371 | $(x\|(y\|z)) = (x\|(z\|y))$ | [337 → 50,342,342,366,342] |
| 376 | $(((((x\|y)\|z)\|y)\|u)\|((x\|y)\|(x\|y))) = (x\|y)$ | [44 → 50,45,45] |
| 420 | $((x\|y)\|z) = (z\|(y\|x))$ | [117 → 371] |
| 430 | $(x\|(y\|z)) = ((z\|y)\|x)$ | [420] |
| 596 | $((x\|y)\|(z\|u)) = ((y\|x)\|(u\|z))$ | [420 → 430] |
| 3288 | $((((A\|B)\|(C\|A))\|D)\|(A\|A)) \neq A \mid \$Ans(OML_Sh)$ | [596 → 133] |
| 8007 | $(((x\|(y\|y))\|x)\|(x\|(y\|z))) = x$ | [103 → 199,4,31] |
| 8087 | $(((x\|y)\|y)\|(y\|((x\|y)\|z))) = y$ | [12 → 8007] |
| 8651 | $((x\|y)\|(x\|(((x\|y)\|x)\|z))) = x$ | [197 → 8087] |
| 12837,12836 | $((x\|(((x\|y)\|x)\|z))\|(y\|x)) = x$ | [420 → 8651] |
| 15661 | $((((x\|y)\|(z\|x))\|u)\|(x\|x)) = x$ | [12836 → 376,12837,12837,12837] |
| 15663 | $\$Ans(OML_Sh)$ | [15661,3288] |

### 3.5.3 A Single Axiom for Modular Ortholattices

The proofs given in this subsection are all in terms of the Sheffer stroke. We have simplified the proof of

MOL-Sh ⇒ {A_SS, B_SS, ONE_SS, MOD_SS}

by presenting it in two stages. First we prove $x|(x|x) = y|(y|y)$, which allows us to introduce a constant we call "1" with the property $x|(x|x) = 1$. We use this in the second stage. The third proof is

$\{$A_SS, B_SS, ONE_SS, MOD_SS$\} \Rightarrow$ MOL-Sh

**Proof MOL-Sh-part1**

| | | |
|---|---|---|
| 1 | $(A\|(A\|A)) \neq ((B\|B)\|B) \ \mid \ \$Ans(ONE_SS)$ | [] |
| 4,3 | $((x\|y)\|(((y\|y)\|z)\|(((((y\|x)\|z)\|z)\|y)\|(y\|u)))) = y$ | [] |
| 5 | $((x\|(y\|z))\|((((y\|z)\|(y\|z))\|u)\|((((((y\|z)\|x)\|u)\|u)\|(y\|z))\|z))) = (y\|z)$ | [3 → 3] |
| 7 | $((x\|y)\|(((y\|y)\|z)\|(((((y\|x)\|z)\|z)\|y)\|(((y\|y)\|u)\|(((((y\|v)\|u)\|u)\|y)\|(y\|w)))))) = y$ | [3 → 5,4,4,4,4,4] |
| 10,9 | $((x\|y)\|(((y\|y)\|z)\|y)) = y$ | [3 → 7] |
| 11 | $(x\|((x\|y)\|(((x\|x)\|z)\|x))) = (((x\|x)\|z)\|x)$ | [9 → 9,10] |
| 16,15 | $((x\|y)\|(y\|y)) = y$ | [9 → 9] |
| 18,17 | $((x\|(((y\|y)\|z)\|y))\|y) = (((y\|y)\|z)\|y)$ | [9 → 9] |
| 19 | $((x\|(y\|z))\|((((y\|z)\|(y\|z))\|(((x\|x)\|u)\|x))\|(((x\|(((x\|x)\|u)\|x))\|(y\|z))\|z))) = (y\|z)$ | [9 → 5] |
| 27 | $((x\|y)\|(((y\|y)\|(((x\|x)\|z)\|x))\|(((x\|(((x\|x)\|z)\|x))\|y)\|(y\|u)))) = y$ | [9 → 3] |
| 33 | $((x\|y)\|(y\|((((y\|y)\|z)\|y)\|(y\|u)))) = y$ | [9 → 3,18] |
| 36,35 | $(((x\|x)\|y)\|x) = (x\|x)$ | [11 → 11,10] |
| 45 | $((x\|y)\|(y\|((y\|y)\|(y\|z)))) = y$ | [33:36] |
| 51 | $((x\|y)\|(((y\|y)\|(x\|x))\|(((x\|(x\|x))\|y)\|(y\|z)))) = y$ | [27:36,36] |
| 59 | $((x\|(y\|z))\|((((y\|z)\|(y\|z))\|(x\|x))\|(((x\|(x\|x))\|(y\|z))\|z))) = (y\|z)$ | [19:36,36] |
| 62,61 | $((x\|(y\|y))\|y) = (y\|y)$ | [17:36,36] |
| 76,75 | $((x\|y)\|(x\|x)) = x$ | [15 → 35,16] |
| 83 | $((x\|x)\|(((x\|x)\|x)\|(((x\|x)\|x)\|(x\|y)))) = x$ | [35 → 3] |
| 88,87 | $(x\|((x\|y)\|(x\|y))) = (x\|y)$ | [75 → 75] |
| 90,89 | $(x\|((y\|x)\|(y\|x))) = (y\|x)$ | [15 → 75] |
| 95 | $((x\|(y\|z))\|((((y\|z)\|(y\|z))\|u)\|((((((y\|z)\|x)\|u)\|u)\|(y\|z))\|y))) = (y\|z)$ | [75 → 3] |
| 97 | $((x\|y)\|(((y\|y)\|((y\|x)\|(y\|x)))\|(((y\|x)\|y)\|(y\|z)))) = y$ | [75 → 3] |
| 108,107 | $((x\|((x\|x)\|(x\|y)))\|(x\|((x\|x)\|(x\|y)))) = x$ | [35 → 45] |
| 125 | $(x\|((x\|x)\|(x\|y))) = (x\|x)$ | [45 → 15,108] |
| 129 | $(((x\|x)\|(y\|z))\|((((y\|z)\|(y\|z))\|x)\|((((x\|x)\|x)\|(y\|z))\|z))) = (y\|z)$ | [61 → 5] |
| 169 | $((x\|y)\|(((y\|y)\|z)\|(((((y\|x)\|z)\|z)\|y)\|(u\|y)))) = y$ | [89 → 3] |
| 179 | $((x\|x)\|((y\|(x\|x))\|((((y\|(x\|x))\|((y\|(x\|x))\|(y\|(x\|x))))\|x)\|(x\|z)))) = x$ | [61 → 51,90] |
| 181 | $((x\|x)\|(((x\|x)\|y)\|(((((x\|x)\|y)\|(((x\|x)\|y)\|((x\|x)\|y)))\|x)\|(x\|z)))) = x$ | [35 → 51,88] |
| 193 | $((x\|(y\|z))\|((((y\|z)\|(y\|z))\|(x\|x))\|(((x\|(x\|x))\|(y\|z))\|y))) = (y\|z)$ | [75 → 51] |
| 198,197 | $((x\|x)\|(x\|((x\|x)\|y))) = x$ | [75 → 125,76] |
| 199 | $(x\|((x\|x)\|(y\|x))) = (x\|x)$ | [89 → 125] |
| 201 | $((x\|y)\|(((x\|y)\|(x\|y))\|x)) = ((x\|y)\|(x\|y))$ | [75 → 125] |
| 217 | $((x\|x)\|(x\|(y\|(x\|x)))) = x$ | [89 → 197] |
| 382,381 | $((x\|x)\|(((x\|x)\|x)\|(((x\|x)\|x)\|(y\|x)))) = x$ | [89 → 83] |
| 449 | $((x\|(y\|z))\|((((y\|z)\|(y\|z))\|(((y\|z)\|x)\|((y\|z)\|x)))\|((((y\|z)\|x)\|(y\|z))\|y))) = (y\|z)$ | [75 → 95] |
| 482,481 | $(((x\|x)\|x)\|(x\|((x\|x)\|y))) = x$ | [97 → 59,76,90,16,76,90,16,76,90,16,16,198,62,76,76,90,16] |
| 488,487 | $((x\|(x\|x))\|((x\|x)\|(x\|y))) = (x\|x)$ | [75 → 481,76] |
| 490,489 | $(((x\|x)\|x)\|(x\|(y\|(x\|x)))) = x$ | [89 → 481] |
| 504,503 | $((x\|(x\|x))\|((x\|x)\|(y\|x))) = (x\|x)$ | [75 → 489,76] |
| 516,515 | $(((x\|y)\|((x\|y)\|(x\|y)))\|(((x\|y)\|(x\|y))\|x)) = ((x\|y)\|(x\|y))$ | [75 → 487] |
| 1107 | $(x\|(y\|(x\|x))) = (x\|x)$ | [217 → 193,76,76,490,16] |
| 1112,1111 | $(((x\|y)\|(x\|y))\|x) = (x\|y)$ | [201 → 193,516,76,62,76] |
| 1114,1113 | $((x\|x)\|(y\|x)) = x$ | [199 → 193,1112,504,76,36,76] |
| 1116,1115 | $(x\|((x\|x)\|y)) = (x\|x)$ | [197 → 193,1114,1112,1114,482,76] |
| 1118,1117 | $((x\|x)\|(x\|y)) = x$ | [125 → 193,1112,488,1114,36,1114] |

| | | |
|---|---|---|
| 1119 | $((x\|y)\|((((((x\|x)\|x)\|y)\|y)\|(x\|x))\|(z\|(x\|x)))) = x$ | [169 → 193,1118,1118,1118,1118,1118,62,76,1118] |
| 1824,1823 | $((((((x\|x)\|x)\|y)\|y)\|(x\|x))\|(((((x\|x)\|x)\|y)\|y)\|(x\|x))) = (x\|x)$ | [1119 → 1107,62] |
| 1830,1829 | $(((((x\|x)\|x)\|y)\|y)\|(x\|x)) = x$ | [1119 → 179,1824,1824,76,1824,76,1824,76,1118] |
| 1834,1833 | $((((x\|(x\|x))\|y)\|y)\|x) = (x\|x)$ | [1117 → 1829,1118] |
| 1835 | $((x\|(x\|x))\|(((x\|x)\|x)\|((x\|x)\|x))) = ((x\|x)\|x)$ | [1829 → 1829] |
| 1856,1855 | $((((((x\|x)\|x)\|y)\|y)\|((((x\|x)\|x)\|y)\|y))\|x) = ((((x\|x)\|x)\|y)\|y)$ | [1829 → 1117] |
| 1859 | $((x\|x)\|((((x\|x)\|x)\|y)\|y)) = x$ | [1829 → 1111,1830,1830] |
| 1886,1885 | $(x\|(((x\|(x\|x))\|y)\|y)) = (x\|x)$ | [1833 → 1111,1834,1118,1834] |
| 1899 | $(x\|(((((x\|x)\|x)\|y)\|y)\|((((x\|x)\|x)\|((((x\|x)\|x)\|y)\|y))\|y))) = ((((x\|x)\|x)\|y)\|y)$ | [1859 → 129,1856] |
| 1961 | $(x\|(((x\|x)\|x)\|(((x\|x)\|x)\|((x\|x)\|x)))) = (x\|x)$ | [1835 → 1885] |
| 1964,1963 | $((((x\|x)\|x)\|(((x\|x)\|x)\|((x\|x)\|x)))\|x) = (x\|x)$ | [1835 → 1833] |
| 2005 | $(((x\|x)\|x)\|(((x\|x)\|x)\|((x\|x)\|x))) = ((x\|x)\|x)$ | [1961 → 449,1964,1964,1118,1964,382,1116,16] |
| 2042,2041 | $(((x\|x)\|x)\|((((x\|x)\|x)\|y)\|y)) = (((x\|x)\|x)\|((x\|x)\|x))$ | [2005 → 1885] |
| 2044,2043 | $(((((x\|x)\|x)\|y)\|y)\|((x\|x)\|x)) = (((x\|x)\|x)\|((x\|x)\|x))$ | [2005 → 1833] |
| 2050,2049 | $((((x\|x)\|x)\|((x\|x)\|x))\|y) = ((x\|x)\|x)$ | [2005 → 181,1886,88] |
| 2063 | $((((x\|x)\|x)\|y)\|y) = ((x\|x)\|x)$ | [1899:2042,2050,2044,90] |
| 2255 | $(((x\|x)\|x)\|(y\|y)) = y$ | [2063 → 15] |
| 2769 | $(x\|(x\|x)) = ((y\|y)\|y)$ | [2255 → 2063] |
| 2770 | $\$Ans(ONE_SS)$ | [2769,1] |

**Proof MOL-Sh-part2**

| | | |
|---|---|---|
| 2 | $((y\|x)\|(((x\|x)\|z)\|(((((x\|y)\|z)\|z)\|x)\|(x\|u)))) = x$ | [] |
| 3 | $(x\|(x\|x)) = 1$ | [] |
| 4 | $((x\|x)\|x) = 1$ | [] |
| 5 | $(A\|((B\|C)\|(B\|C))) \neq (B\|((A\|C)\|(A\|C)))$ \| $\$Ans(A_SS)$ | [] |
| 6 | $((A\|A)\|(A\|B)) \neq A$ \| $\$Ans(B_SS)$ | [] |
| 7 | $(A\|(B\|(A\|(C\|C)))) \neq (A\|(C\|(A\|(B\|B)))$ \| $\$Ans(MOD_SS)$ | [] |
| 258 | $(x\|(x\|x)) = (y\|(y\|y))$ | [3 → 3] |
| 259 | $(x\|(x\|x)) = ((y\|y)\|y)$ | [4 → 3] |
| 260 | $((x\|y)\|(y\|(((((y\|x)\|(((y\|y)\|z)\|(((((y\|y)\|z)\|z)\|y)\|(y\|u))))\|(((y\|y)\|z)\|(((((y\|y)\|z)\|z)\|y)\|(y\|u))))\|y)\|(y\|v)))) = y$ | [2 → 2] |
| 261 | $((x\|(y\|z))\|((((y\|z)\|(y\|z))\|u)\|((((((y\|z)\|x)\|u)\|u)\|(y\|z))\|z))) = (y\|z)$ | [2 → 2] |
| 262 | $((x\|y)\|(y\|y)) = y$ | [260 → 260] |
| 263 | $((x\|y)\|(((y\|y)\|z)\|y)) = y$ | [260 → 2] |
| 264 | $((x\|(x\|x))\|(y\|y)) = y$ | [259 → 262] |
| 265 | $((x\|(y\|y))\|y) = (y\|y)$ | [262 → 262] |
| 266 | $(((x\|x)\|(y\|x))\|((((y\|x)\|(y\|x))\|z)\|((((x\|z)\|z)\|(y\|x))\|x))) = (y\|x)$ | [262 → 261] |
| 267 | $(((x\|x)\|(y\|x))\|((((y\|x)\|(y\|x))\|z)\|((((x\|z)\|z)\|(y\|x))\|((y\|x)\|u)))) = (y\|x)$ | [262 → 2] |
| 268 | $((x\|(x\|x))\|y) = (y\|y)$ | [262 → 264] |
| 269 | $((x\|y)\|(((y\|y)\|((y\|x)\|(y\|x)))\|(((y\|x)\|y)\|(y\|z)))) = y$ | [264 → 2] |
| 270 | $(1\|x) = (x\|x)$ | [4 → 265] |
| 271 | $((x\|y)\|((z\|(y\|y))\|y)) = y$ | [265 → 262] |
| 272 | $((x\|(((y\|y)\|z)\|y))\|y) = (((y\|y)\|z)\|y)$ | [263 → 263] |
| 273 | $((x\|(y\|y))\|((z\|y)\|(y\|y))) = (y\|y)$ | [262 → 271] |
| 274 | $(((x\|x)\|y)\|x) = (x\|x)$ | [263 → 272] |
| 275 | $(((1\|x)\|y)\|x) = (x\|x)$ | [270 → 274] |
| 276 | $((x\|y)\|(x\|x)) = ((x\|x)\|(x\|x))$ | [262 → 274] |
| 277 | $((x\|x)\|(((x\|x)\|x)\|(((x\|x)\|x)\|(x\|y)))) = x$ | [274 → 2] |
| 279 | $((x\|y)\|(x\|x)) = x$ | [262 → 276] |

| | | |
|---|---|---|
| 280 | $(x\|((x\|y)\|(x\|y))) = (x\|y)$ | [279 → 279] |
| 281 | $(x\|((y\|x)\|(y\|x))) = (y\|x)$ | [271 → 279] |
| 282 | $((x\|y)\|(1\|x)) = x$ | [270 → 279] |
| 283 | $((x\|y)\|((z\|(x\|x))\|x)) = x$ | [265 → 279] |
| 284 | $(x\|(1\|(x\|y))) = (x\|y)$ | [282 → 282] |
| 285 | $(x\|(1\|(y\|x))) = (y\|x)$ | [271 → 282] |
| 286 | $(x\|(((y\|y)\|y)\|(z\|x))) = (z\|x)$ | [4 → 285] |
| 287 | $(x\|((y\|(y\|y))\|(z\|x))) = (z\|x)$ | [3 → 285] |
| 288 | $((x\|y)\|(((y\|y)\|z)\|(((((y\|x)\|z)\|z)\|y)\|(u\|y)))) = y$ | [285 → 2] |
| 289 | $((((x\|y)\|(x\|y))\|x)\|(((x\|x)\|z)\|(((((x\|y)\|z)\|z)\|x)\|(x\|u)))) = x$ | [280 → 2] |
| 290 | $(x\|((((y\|x)\|(y\|x))\|z)\|(y\|x))) = (y\|x)$ | [274 → 281] |
| 291 | $(x\|x) = (1\|x)$ | [282 → 287] |
| 292 | $((x\|x)\|((y\|(y\|y))\|(((x\|x)\|x)\|(x\|z)))) = x$ | [259 → 277] |
| 293 | $((x\|x)\|(((x\|x)\|x)\|(((x\|x)\|x)\|(y\|x)))) = x$ | [290 → 277] |
| 295 | $((x\|x)\|((y\|(y\|y))\|((z\|(z\|z))\|(x\|u)))) = x$ | [259 → 292] |
| 296 | $((x\|x)\|(y\|(y\|y))) = x$ | [258 → 293] |
| 297 | $((x\|x), 1) = x$ | [3 → 293] |
| 298 | $(x, 1) = (x\|x)$ | [279 → 297] |
| 299 | $(x\|(1, 1)) = 1$ | [297 → 262] |
| 300 | $(x\|(y\|(y\|y))) = (x\|x)$ | [3 → 298] |
| 301 | $(((1, 1)\|x)\|((1, 1)\|x)) = (1, 1)$ | [275 → 298] |
| 302 | $((x\|(y\|(y\|y)))\|(z\|(z\|z))) = x$ | [300 → 296] |
| 303 | $((1, 1)\|(1, 1)) = ((1, 1)\|x)$ | [301 → 280] |
| 305 | $((1, 1)\|x) = 1$ | [299 → 303] |
| 308 | $((x\|x)\|(x\|y)) = x$ | [283 → 295] |
| 309 | $\$Ans(B_SS)$ | [308,6] |
| 310 | $(x\|((x\|x)\|y)) = (x\|x)$ | [308 → 308] |
| 311 | $((x\|(y\|(y\|y)))\|(x\|z)) = x$ | [300 → 308] |
| 312 | $((x, 1)\|(x\|y)) = x$ | [298 → 308] |
| 313 | $((1\|x)\|(x\|y)) = x$ | [291 → 308] |
| 314 | $((x\|x)\|(y\|x)) = x$ | [290 → 308] |
| 315 | $(((x\|y)\|(x\|y))\|x) = (x\|y)$ | [283 → 308] |
| 316 | $(x\|x) = (x, 1)$ | [312 → 280] |
| 317 | $((1\|(x\|y))\|x) = (x\|y)$ | [283 → 313] |
| 318 | $(x\|(y\|(x\|x))) = (x\|x)$ | [314 → 314] |
| 319 | $((x\|(y\|(y\|y)))\|(z\|x)) = x$ | [300 → 314] |
| 320 | $((x\|x)\|(x\|y)) = ((x\|x)\|(x\|x))$ | [314 → 310] |
| 321 | $((x\|y)\|(y\|((y\|y)\|z))) = y$ | [310 → 262] |
| 322 | $((x\|((x\|x)\|y))\|(x\|z)) = x$ | [310 → 308] |
| 323 | $(((x\|(x\|x))\|(y\|z))\|y) = (y\|z)$ | [3 → 317] |
| 324 | $(x\|(y\|((z\|(z\|z))\|x))) = (x\|x)$ | [268 → 318] |
| 325 | $(1\|(x\|((x\|x)\|y))) = x$ | [305 → 321] |
| 326 | $(1\|(x\|(y\|(x\|x)))) = x$ | [290 → 325] |
| 327 | $(1\|((x\|x)\|(y\|x))) = (x\|x)$ | [314 → 326] |
| 328 | $(((x\|x)\|x)\|((y\|y)\|(z\|y))) = (y\|y)$ | [4 → 327] |
| 329 | $((x\|x)\|(x\|y)) = ((x\|x)\|(x\|z))$ | [320 → 320] |
| 331 | $((x\|y)\|(y\|(((((y\|x)\|(z\|y))\|(z\|y))\|y)\|(u\|y)))) = y$ | [314 → 288] |
| 332 | $((x\|y)\|(y\|(((((y\|x)\|(y\|z))\|(y\|z))\|y)\|(u\|y)))) = y$ | [308 → 288] |
| 333 | $((x\|y)\|(1\|(((((y\|x)\|y)\|y)\|y)\|(z\|y)))) = y$ | [4 → 288] |
| 334 | $(((((x\|y)\|x)\|x)\|x)\|(y\|x)) = x$ | [286 → 288] |
| 335 | $((x\|y)\|((((y\|x)\|(z\|y))\|(z\|y))\|y)) = y$ | [281 → 331] |
| 336 | $((x\|y)\|((((x\|((x\|y)\|(x\|y)))\|(z\|x))\|(z\|x))\|x)) = x$ | [315 → 335] |
| 337 | $((x\|y)\|((((y\|x)\|(y\|z))\|(y\|z))\|y)) = y$ | [281 → 332] |
| 338 | $((x\|y)\|((((x\|((x\|y)\|(x\|y)))\|(x\|z))\|(x\|z))\|x)) = x$ | [315 → 337] |

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| 339 | $(x\|(((((y\|x)\|(x\|x))\|((y\|x)\|z))\|((y\|x)\|z))\|(y\|x))) = (y\|x)$ | [314 → 337] |
| 340 | $(x\|(((((x\|y)\|(x\|x))\|((x\|y)\|z))\|((x\|y)\|z))\|(x\|y))) = (x\|y)$ | [308 → 337] |
| 341 | $((x\|y)\|((((x\|y)\|(z\|x))\|(z\|x))\|x)) = x$ | [280 → 336] |
| 342 | $((x\|y)\|((((x\|y)\|(x\|z))\|(x\|z))\|x)) = x$ | [280 → 338] |
| 343 | $((x\|y)\|((x\|y)\|x)) = x$ | [302 → 342] |
| 344 | $((x,1)\|(1\|(((1\|x),1)\|(1\|y)))) = 1$ | [305 → 269] |
| 345 | $((x,1)\|(1\|(((1\|x),1)\|y))) = 1$ | [326 → 344] |
| 346 | $((x,1)\|(1\|(((x\|x),1)\|y))) = 1$ | [291 → 345] |
| 347 | $((x,1)\|(1\|(x\|y))) = 1$ | [297 → 346] |
| 348 | $((x\|x)\|(1\|(x\|y))) = 1$ | [316 → 347] |
| 349 | $((x\|x)\|((x\|y)\|(x\|y))) = 1$ | [291 → 348] |
| 350 | $(((x\|y)\|(x\|y))\|(y\|y)) = 1$ | [273 → 349] |
| 352 | $(x\|(((x\|((y\|x)\|z))\|((y\|x)\|z))\|(y\|x))) = (y\|x)$ | [262 → 339] |
| 353 | $(x\|(((x\|y)\|((y\|x)\|((((y\|x)\|(z\|y))\|(z\|y))\|y)))\|(y\|x))) = (y\|x)$ | [341 → 352] |
| 354 | $(x\|(x\|(y\|x))) = (y\|x)$ | [302 → 352] |
| 355 | $(x\|(((x\|((x\|y)\|z))\|((x\|y)\|z))\|(x\|y))) = (x\|y)$ | [279 → 340] |
| 356 | $(x\|(x\|(x\|y))) = (x\|y)$ | [302 → 355] |
| 357 | $((x\|y)\|(((x\|x)\|z)\|(((((x\|y)\|z)\|z)\|x)\|(x\|u)))) = x$ | [315 → 289] |
| 358 | $((x\|y)\|(((x\|x),1)\|(((((x\|y)\|(x\|y)),1)\|x)\|(x\|z)))) = x$ | [316 → 357] |
| 359 | $((x\|y)\|(x\|(((((x\|y)\|(x\|y)),1)\|x)\|(x\|z)))) = x$ | [297 → 358] |
| 360 | $((x\|y)\|(x\|(((x\|y)\|x)\|(x\|z)))) = x$ | [297 → 359] |
| 361 | $(x\|(((x\|y)\|y)\|(y\|x))) = (y\|x)$ | [342 → 353] |
| 362 | $(((x\|x)\|(y\|x))\|((((y\|x)\|(y\|x))\|(x\|x))\|((x\|(y\|x))\|x))) = (y\|x)$ | [279 → 266] |
| 363 | $(x\|((((y\|x)\|(y\|x))\|(x\|x))\|((x\|(y\|x))\|x))) = (y\|x)$ | [314 → 362] |
| 364 | $(x\|(1\|((x\|(y\|x))\|x))) = (y\|x)$ | [350 → 363] |
| 366 | $((x\|(y\|x))\|x) = (y\|x)$ | [285 → 364] |
| 367 | $(((x\|(x\|x))\|(y\|z))\|y) = ((y\|(y\|z))\|y)$ | [323 → 366] |
| 368 | $((x\|(y\|x))\|(x\|((y\|x)\|(x\|z)))) = x$ | [366 → 360] |
| 369 | $((x\|(y\|x))\|(y\|x)) = x$ | [366 → 343] |
| 373 | $((x\|(x\|y))\|x) = (x\|y)$ | [323 → 367] |
| 374 | $((x\|y)\|x) = (x\|(x\|y))$ | [356 → 373] |
| 375 | $((x\|y)\|y) = (y\|(x\|y))$ | [354 → 373] |
| 376 | $((x\|(x\|y))\|(x\|y)) = x$ | [373 → 343] |
| 377 | $(((x\|y)\|x)\|x) = (x\|y)$ | [374 → 373] |
| 378 | $(x\|((y\|(x\|y))\|(y\|x))) = (y\|x)$ | [375 → 361] |
| 380 | $((x\|y)\|(((y\|y)\|((y\|x)\|z))\|(((y\|x)\|y)\|(y\|u)))) = y$ | [376 → 2] |
| 382 | $((x\|y)\|(1\|(((y\|x)\|y)\|(z\|y)))) = y$ | [377 → 333] |
| 383 | $(((x\|y)\|x)\|(y\|x)) = x$ | [377 → 334] |
| 386 | $(((x\|(x\|(x\|y)))\|x)\|(y\|x)) = x$ | [356 → 383] |
| 387 | $((x\|(x\|y))\|(y\|x)) = x$ | [374 → 383] |
| 389 | $((x\|y)\|(1\|((y\|(y\|x))\|(z\|y)))) = y$ | [374 → 382] |
| 390 | $((x\|y)\|(1\|((x\|(x\|(1\|(x\|y))))\|(z\|x)))) = x$ | [317 → 389] |
| 391 | $((x\|y)\|(1\|((y\|(y\|x))\|(y\|z)))) = y$ | [377 → 389] |
| 392 | $((1\|((x\|(x\|y))\|(x\|z)))\|(((y\|x)\|(1\|((x\|(x\|y))\|(x\|z))))\|((y\|x)\|(1\|((x\|(x\|y))\|(x\|z))))))) = x$ | [281 → 391] |
| 393 | $((x\|y)\|(1\|((x\|(x\|y))\|(z\|x)))) = x$ | [284 → 390] |
| 395 | $((x\|x)\|(((x\|x)\|y)\|(((x\|x)\|x)\|(x\|z)))) = x$ | [356 → 380] |
| 397 | $((x\|x)\|(((x\|x)\|y)\|((z\|(z\|z))\|(x\|u)))) = x$ | [259 → 395] |
| 400 | $(x\|((((y\|x)\|(y\|x))\|z)\|((((x\|z)\|z)\|(y\|x))\|((y\|x)\|u)))) = (y\|x)$ | [314 → 267] |
| 401 | $(x\|((y\|x)\|((((x\|y)\|y)\|(y\|x))\|((y\|x)\|z)))) = (y\|x)$ | [315 → 400] |
| 402 | $((1\|((x\|(x\|y))\|(x\|z)))\|(x\|((y\|x)\|(1\|((x\|(x\|y))\|(x\|z))))))) = x$ | [391 → 392] |
| 403 | $((1\|((x\|(x\|y))\|(x\|z)))\|(x\|x)) = x$ | [391 → 402] |
| 405 | $((1\|((x\|y)\|(y\|z)))\|(y\|y)) = y$ | [354 → 403] |
| 406 | $((1\|((x\|y)\|(z\|y)))\|(y\|y)) = y$ | [401 → 405] |

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| 407 | $(((x\|y)\|(y\|z))\|((x\|y)\|(y\|z))) = (y\|((x\|y)\|(y\|z)))$ | [405 → 275] |
| 408 | $((1\|((x\|y)\|(z\|x)))\|(x\|x)) = x$ | [377 → 406] |
| 409 | $(((x\|y)\|(z\|x))\|((x\|y)\|(z\|x))) = (x\|((x\|y)\|(z\|x)))$ | [408 → 275] |
| 410 | $((x\|y)\|(x\|((z\|x)\|(x\|y)))) = ((z\|x)\|(x\|y))$ | [407 → 281] |
| 411 | $((x\|((y\|z)\|(z\|u)))\|(z\|((y\|z)\|(z\|u)))) = ((y\|z)\|(z\|u))$ | [407 → 262] |
| 412 | $(x\|((x\|y)\|((z\|(x\|y))\|((x\|y)\|(1\|((x\|(x\|y))\|(u\|x)))))))) = ((z\|(x\|y))\|((x\|y)\|(1\|((x\|(x\|y))\|(u\|x)))))$ | [393 → 410] |
| 414 | $((x\|y)\|(y\|((y\|z)\|(x\|y)))) = ((y\|z)\|(x\|y))$ | [409 → 281] |
| 417 | $((x\|((y\|((z\|u)\|(u\|v)))\|(u\|((z\|u)\|(u\|v)))))\|(u\|((z\|u)\|(u\|v)))) = ((z\|u)\|(u\|v))$ | [411 → 411] |
| 420 | $((x\|(y\|z))\|((y\|z)\|(1\|((y\|(y\|z))\|(u\|y))))) = (y\|((y\|z)\|((x\|(y\|z))\|y)))$ | [393 → 412] |
| 421 | $(x\|((x\|y)\|((z\|(x\|y))\|x))) = ((z\|(x\|y))\|x)$ | [393 → 420] |
| 422 | $(((x\|(x\|y))\|(y\|x))\|y) = (y\|((y\|x)\|(x\|y)))$ | [387 → 421] |
| 423 | $(x\|((x\|y)\|(y\|x))) = (y\|x)$ | [387 → 422] |
| 424 | $((x\|y)\|(x\|y)) = ((y\|x)\|(x\|y))$ | [423 → 414] |
| 425 | $(x\|((y\|x)\|(x\|y))) = (x\|y)$ | [424 → 280] |
| 426 | $(((x\|y)\|(y\|x))\|((y\|x)\|z)) = (y\|x)$ | [424 → 308] |
| 427 | $((x\|(y\|x))\|(x\|y)) = x$ | [425 → 368] |
| 428 | $(((x\|y)\|y)\|(y\|x)) = y$ | [375 → 427] |
| 429 | $(x\|y) = (y\|x)$ | [427 → 378] |
| 430 | $((x\|y)\|(((y\|y)\|z)\|(((((x\|y)\|z)\|z)\|y)\|(y\|u)))) = y$ | [429 → 2] |
| 431 | $(x\|((y\|x)\|(y\|((y\|x)\|z)))) = (y\|x)$ | [428 → 401] |
| 432 | $(((x\|y)\|(y\|x))\|((x\|y)\|z)) = (y\|x)$ | [429 → 426] |
| 433 | $(x\|((y\|x)\|(y\|(z\|(y\|x))))) = (y\|x)$ | [431 → 431] |
| 434 | $((((x\|y)\|z)\|(u\|(u\|u)))\|(y\|x)) = ((x\|y)\|z)$ | [432 → 319] |
| 435 | $(x\|((y\|z)\|(z\|x))) = (z\|x)$ | [410 → 433] |
| 439 | $(x\|((y\|z)\|(x\|z))) = (z\|x)$ | [429 → 435] |
| 440 | $(x\|((y\|y)\|(y\|z))) = (y\|x)$ | [329 → 435] |
| 441 | $(x\|(y\|(x\|(z\|y)))) = ((z\|y)\|x)$ | [386 → 439] |
| 442 | $(x\|((x\|y)\|(z\|y))) = (y\|x)$ | [429 → 439] |
| 445 | $(x\|((x\|(y\|z))\|z)) = ((y\|z)\|x)$ | [386 → 442] |
| 446 | $(((x\|y)\|(z\|y))\|x) = (y\|x)$ | [429 → 442] |
| 448 | $(((x\|(y\|z))\|z)\|x) = ((y\|z)\|x)$ | [386 → 446] |
| 451 | $(((x\|(x\|x))\|y)\|z) = (z\|(y\|y))$ | [324 → 441] |
| 452 | $(x\|(y\|(x\|(z\|y)))) = (x\|(z\|y))$ | [429 → 441] |
| 453 | $((x\|((y\|(y\|y))\|(z\|u)))\|(z\|z)) = z$ | [397 → 445] |
| 454 | $((((x\|y)\|z)\|y)\|z) = ((x\|y)\|z)$ | [429 → 448] |
| 455 | $(((x\|(y\|z))\|z)\|x) = ((z\|y)\|x)$ | [429 → 448] |
| 456 | $((1\|x)\|y) = (y\|(x\|x))$ | [349 → 451] |
| 457 | $((x, 1)\|y) = (y\|(x\|x))$ | [429 → 456] |
| 458 | $((1\|x)\|y) = ((x\|x)\|y)$ | [429 → 456] |
| 459 | $(x\|((y\|(z\|(z\|z)))\|(y\|u))) = (x\|((y\|u)\|(x\|y)))$ | [311 → 452] |
| 460 | $((x\|(y\|(z\|x)))\|y) = (y\|(z\|x))$ | [429 → 452] |
| 461 | $((x\|(y\|(z\|x)))\|y) = ((x\|z)\|y)$ | [429 → 455] |
| 463 | $(x\|(y\|z)) = ((z\|y)\|x)$ | [460 → 461] |
| 464 | $((x\|y)\|z) = (z\|(y\|x))$ | [461 → 460] |
| 465 | $(x\|(y\|(z\|u))) = ((y\|(u\|z))\|x)$ | [463 → 463] |
| 466 | $(x\|(y\|(z, 1))) = ((y\|(z\|z))\|x)$ | [457 → 463] |
| 467 | $(x\|(y\|z)) = (x\|(z\|y))$ | [429 → 463] |
| 468 | $(x\|((y\|x)\|(y\|z))) = (y\|x)$ | [463 → 435] |
| 469 | $(((x\|y)\|(x\|y))\|(z\|(y\|x))) = (x\|y)$ | [463 → 308] |
| 470 | $((x\|(y\|z))\|u) = (u\|(x\|(z\|y)))$ | [464 → 464] |
| 471 | $(((1\|x)\|y)\|z) = (z\|(y\|(x\|x)))$ | [458 → 464] |
| 472 | $(((x\|y)\|z)\|u) = (u\|(z\|(y\|x)))$ | [467 → 464] |
| 473 | $((x\|(y\|z))\|u) = (u\|((z\|y)\|x))$ | [467 → 464] |

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| 474 | $(x\|(((y\|z)\|x)\|(u\|(z\|y)))) = ((y\|z)\|x)$ | [464 → 468] |
| 477 | $(((x\|(y\|z))\|u)\|v) = (v\|((x\|(z\|y))\|u))$ | [465 → 464] |
| 478 | $((x\|((y\|z)\|(z\|u)))\|(z\|z)) = z$ | [417 → 453] |
| 481 | $((x\|((y\|z)\|(y\|u)))\|(y\|y)) = y$ | [429 → 478] |
| 484 | $((((x\|y)\|(z\|y))\|u)\|(y\|y)) = y$ | [472 → 478] |
| 486 | $(((x\|x)\|y)\|((z\|x)\|(z\|x))) = (z\|x)$ | [328 → 484] |
| 488 | $((x\|x)\|(y\|((z\|x)\|(u\|x)))) = x$ | [472 → 484] |
| 490 | $(((x\|(y\|z))\|(x\|(y\|z)))\|(u\|((z\|y)\|(v\|(x\|(y\|z))))))) = (x\|(y\|z))$ | [469 → 488] |
| 492 | $(x\|((((y\|y)\|z)\|x)\|(u\|y))) = (((y\|y)\|z)\|x)$ | [486 → 468] |
| 493 | $((((x\|((y\|z)\|(y\|u)))\|(y\|y))\|(v\|(v\|v)))\|(((y\|z)\|(y\|u))\|x)) = y$ | [481 → 434] |
| 494 | $(((x\|x)\|y)\|((x\|z)\|(x\|z))) = (x\|z)$ | [469 → 492] |
| 495 | $(x\|(((y\|y)\|z)\|((y\|u)\|x))) = ((y\|u)\|x)$ | [492 → 474] |
| 496 | $((((x\|x)\|y)\|(((x\|x)\|y)\|((x\|z)\|(x\|z))))\|(x\|z)) = ((x\|x)\|y)$ | [494 → 376] |
| 497 | $(x\|((y\|z)\|(x\|y))) = (x\|y)$ | [322 → 459] |
| 498 | $(((x\|y)\|x)\|(((y\|x)\|z)\|x)) = (((x\|y)\|x)\|(y\|x))$ | [383 → 497] |
| 499 | $(x\|((x\|y)\|(y\|z))) = (x\|y)$ | [429 → 497] |
| 500 | $(((x\|y)\|(y\|z))\|x) = (x\|y)$ | [464 → 497] |
| 502 | $((((x\|y)\|z)\|(z\|u))\|(y\|x)) = ((x\|y)\|z)$ | [464 → 499] |
| 503 | $(((x\|y)\|x)\|(((y\|x)\|z)\|x)) = x$ | [383 → 498] |
| 506 | $((((x\|x)\|y)\|(x\|z))\|(x\|z)) = ((x\|x)\|y)$ | [494 → 496] |
| 508 | $((x\|((y\|x)\|(x\|z)))\|(u\|((z\|x)\|(v\|((y\|x)\|(x\|z))))))) = ((y\|x)\|(x\|z))$ | [407 → 490] |
| 509 | $((x\|(((((y\|x)\|z)\|z)\|x)\|(x\|y)))\|(u\|x)) = (((((y\|x)\|z)\|z)\|x)\|(x\|y))$ | [430 → 508] |
| 510 | $(((((x\|y)\|z)\|z)\|y)\|(y\|x)) = y$ | [478 → 509] |
| 511 | $(((x\|(x\|(y\|z)))\|z)\|(z\|y)) = z$ | [464 → 510] |
| 512 | $((x\|(y\|(y\|(z\|x))))\|(x\|z)) = x$ | [472 → 510] |
| 513 | $(((x\|(x\|(y\|z)))\|z)\|(y\|z)) = z$ | [429 → 511] |
| 514 | $((x\|y)\|((z\|(z\|(x\|y)))\|y)) = y$ | [464 → 511] |
| 516 | $(((x\|((y\|z)\|x))\|y)\|(z\|y)) = y$ | [464 → 513] |
| 518 | $(x\|((y\|(y\|((x\|(z\|(z\|(u\|x))))\|(x\|u))))\|(x\|u))) = (x\|u)$ | [512 → 514] |
| 519 | $((x\|y)\|((z\|((y\|x)\|z))\|y)) = y$ | [464 → 514] |
| 520 | $((x\|y)\|((z\|((u\|((y\|x)\|u))\|y))\|((x\|y)\|((u\|((y\|x)\|u))\|y)))) = y$ | [439 → 516] |
| 521 | $(x\|((y\|(y\|x))\|(x\|z))) = (x\|z)$ | [512 → 518] |
| 523 | $((x\|y)\|((z\|((u\|((y\|x)\|u))\|y))\|y)) = y$ | [519 → 520] |
| 524 | $((x\|((y\|((z\|(x\|x))\|y))\|z))\|z) = (z\|z)$ | [523 → 495] |
| 525 | $((((x\|(x\|(y\|(z\|z))))\|y)\|z)\|y) = (y\|y)$ | [477 → 524] |
| 526 | $(((x\|((y\|(z\|z))\|x))\|y)\|z) = ((y\|y)\|z)$ | [524 → 448] |
| 527 | $(((x\|(x\|(y\|(z\|z))))\|y)\|z) = ((y\|y)\|z)$ | [525 → 454] |
| 528 | $(((x\|((y\|(z\|z))\|x))\|((y\|(z\|z))\|x))\|z) = ((((y\|(z\|z))\|x)\|((y\|(z\|z))\|x))\|z)$ | [454 → 526] |
| 529 | $((x\|(x\|(y\|y)))\|y) = ((x\|x)\|y)$ | [500 → 526] |
| 530 | $((x\|(((y\|(y\|y))\|z)\|x))\|z) = ((x\|x)\|z)$ | [451 → 529] |
| 531 | $((((((x\|x)\|y)\|(x\|x))\|(x\|x))\|((y\|(x\|x))\|z))\|x) = ((((y\|(x\|x))\|z)\|((y\|(x\|x))\|z))\|x)$ | [503 → 527] |
| 532 | $((((x\|(y\|y))\|z)\|((x\|(y\|y))\|z))\|y) = (z\|y)$ | [369 → 528] |
| 533 | $((((x\|(y\|y))\|z)\|((x\|(y\|y))\|z))\|y) = ((((y\|y)\|x)\|((x\|(y\|y))\|z))\|y)$ | [506 → 531] |
| 534 | $((((x\|x)\|y)\|((y\|(x\|x))\|z))\|x) = (z\|x)$ | [532 → 533] |
| 536 | $((((x\|x)\|y)\|(z\|(y\|(x\|x))))\|x) = (z\|x)$ | [470 → 534] |
| 537 | $((((x\|x)\|y)\|(z\|(y\|(x\|x))))\|x) = (((z\|(y\|(x\|x)))\|(z\|u))\|x)$ | [468 → 534] |
| 539 | $(((x\|(y\|(z\|z)))\|(x\|u))\|z) = (x\|z)$ | [536 → 537] |
| 540 | $(((x\|(y\|z))\|(x\|u))\|(z\|z)) = (x\|(z\|z))$ | [314 → 539] |
| 541 | $(((x\|y)\|(x\|z))\|((u\|y)\|(v\|y))) = (x\|((u\|y)\|(v\|y)))$ | [488 → 539] |
| 542 | $(((x\|y)\|(x\|((z\|z)\|u)))\|z) = (x\|z)$ | [470 → 539] |
| 544 | $(((x\|y)\|(x\|(z\|u)))\|(z\|z)) = (x\|(z\|z))$ | [314 → 542] |
| 545 | $((x\|x)\|((y\|z)\|(z\|(u\|x)))) = (z\|(x\|x))$ | [473 → 540] |
| 546 | $((x\|x)\|(((y\|x)\|(z\|u))\|z)) = ((z\|u)\|(x\|x))$ | [502 → 545] |
| 547 | $((x\|x)\|((y\|z)\|(z\|(u\|x)))) = ((x\|x)\|z)$ | [464 → 545] |

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| 548 | $(((x,1)\|(x,1))\|((y\|z)\|((u\|(x\|x))\|z))) = (((x,1)\|(x,1))\|z)$ | [466 → 547] |
| 549 | $(((x\|y)\|(z\|y))\|((u\|y)\|(u\|v))) = (u\|((z\|y)\|(x\|y)))$ | [464 → 541] |
| 550 | $(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$ | [541 → 549] |
| 551 | $\$Ans(A_SS)$ | [550,5] |
| 552 | $(x\|((y\|z)\|((u\|(x\|x))\|z))) = (((x,1)\|(x,1))\|z)$ | [312 → 548] |
| 553 | $(x\|((y\|z)\|((u\|(x\|x))\|z))) = (x\|z)$ | [312 → 552] |
| 554 | $(((x\|y)\|(x\|z))\|((u\|v)\|(x\|v))) = (((x\|y)\|(x\|z))\|v)$ | [493 → 553] |
| 555 | $(((x\|y)\|(x\|((x\|z)\|u)))\|z) = (x\|((x\|z)\|(x\|z)))$ | [544 → 554] |
| 556 | $(((x\|y)\|(x\|((x\|z)\|u)))\|z) = (z\|x)$ | [442 → 555] |
| 559 | $((x\|(x\|y))\|(y\|((y\|x)\|z))) = ((x\|x)\|(y\|((y\|x)\|z)))$ | [556 → 530] |
| 560 | $(x\|((y\|y)\|(x\|((x\|y)\|z)))) = (x\|((x\|y)\|z))$ | [559 → 521] |
| 561 | $(x\|((y\|y)\|(((x\|y)\|z)\|x))) = (x\|((x\|y)\|z))$ | [429 → 560] |
| 562 | $(x\|((x\|y)\|(z\|z))) = (x\|((x\|z)\|(x\|y)))$ | [546 → 561] |
| 563 | $(x\|((x\|(y\|y))\|(x\|z))) = (x\|(y\|(x\|z)))$ | [440 → 562] |
| 564 | $(x\|(((1\|y)\|x)\|(x\|z))) = (x\|(y\|(x\|z)))$ | [456 → 563] |
| 565 | $(x\|(((1\|y)\|x)\|(x\|(z\|z)))) = (x\|(z\|(x\|(y\|y))))$ | [471 → 563] |
| 566 | $(x\|(y\|(x\|(z\|z)))) = (x\|(z\|(x\|(y\|y))))$ | [564 → 565] |
| 567 | $\$Ans(MOD_SS)$ | [566,7] |

**Proof MOL-Sh-sound**

| | | |
|---|---|---|
| 1 | $x = x$ | [] |
| 2 | $(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$ | [] |
| 4,3 | $((x\|x)\|(x\|y)) = x$ | [] |
| 6 | $(x\|(y\|(x\|(z\|z)))) = (x\|(z\|(x\|(y\|y))))$ | [] |
| 7 | $(x\|(x\|x)) = 1$ | [] |
| 9 | $((B\|A)\|(((A\|A)\|C)\|(((((A\|B)\|C)\|C)\|A)\|(A\|D)))) \neq A \mid \$Ans(MOL_Sh)$ | [] |
| 11,10 | $(x\|((x\|x)\|y)) = (x\|x)$ | [3 → 3] |
| 12 | $((x\|x)\|(y\|((x\|z)\|(x\|z)))) = x$ | [2 → 3] |
| 16 | $(x\|((y\|x)\|(y\|x))) = (y\|x)$ | [3 → 2] |
| 18 | $((x\|x)\|((y\|(x\|z))\|(y\|(x\|z)))) = (y\|(x\|x))$ | [3 → 2,4] |
| 20 | $((x\|x)\|x) = 1$ | [3 → 7] |
| 22 | $(x\|(((x\|y)\|y)\|((x\|y)\|y))) = 1$ | [2 → 7] |
| 25,24 | $((x\|x),1) = x$ | [7 → 3] |
| 28 | $(x,1) = (x\|x)$ | [7 → 10] |
| 31 | $(x\|x) = (x,1)$ | [28] |
| 34 | $(x\|(y\|(x\|z))) = (x\|((z\|z)\|(x\|(y\|y))))$ | [3 → 6] |
| 36,35 | $(x\|(y\|(x\|x))) = (x\|x)$ | [10 → 6,4] |
| 37 | $(x\|(y,1)) = (x\|(x\|(x\|(y\|y))))$ | [7 → 6] |
| 38 | $((x\|x)\|(y\|x)) = x$ | [3 → 6,11,4] |
| 41 | $(x\|((y\|y)\|(x\|(z\|z)))) = (x\|(z\|(x\|y)))$ | [34] |
| 58 | $(x\|(y\|(x\|(x\|(y\|y))))) = (x,1)$ | [20 → 6,4] |
| 66 | $(x\|((y,1)\|(y\|y))) = (y\|((x,1)\|(x,1)))$ | [28 → 2] |
| 68 | $(x\|((y,1)\|(y,1))) = (y\|((x,1)\|(x\|x)))$ | [66] |
| 73 | $(x\|(y\|(x\|(z,1)))) = (x\|(z\|(x\|(y\|y))))$ | [31 → 6] |
| 85,84 | $((x,1)\|(x\|y)) = x$ | [31 → 3] |
| 88 | $(x\|y) = (y\|x)$ | [68:85,85] |
| 91,90 | $((x\|y)\|(y\|y)) = y$ | [38 → 88] |
| 92 | $(x\|x) = (1\|x)$ | [28 → 88] |
| 94,93 | $(1\|(x\|x)) = x$ | [24 → 88] |
| 95 | $(((x\|x)\|y)\|x) = (x\|x)$ | [10 → 88] |
| 100,99 | $((x\|y)\|(x\|x)) = x$ | [3 → 88] |

| | | |
|---|---|---|
| 101 | $(x\|((y\|z)\|(y\|z))) = (((x\|z)\|(x\|z))\|y)$ | [2 → 88] |
| 102 | $(1\|x) = (x\|x)$ | [92] |
| 103 | $(((x\|y)\|(x\|y))\|z) = (x\|((z\|y)\|(z\|y)))$ | [101] |
| 104 | $((((A\|A)\|C)\|(((((A\|B)\|C)\|C)\|A)\|(A\|D)))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$ | [88 → 9] |
| 122 | $((x,1)\|(y\|((x\|z)\|(x\|z)))) = x$ | [31 → 12] |
| 146 | $(x\|((y\|z)\|(x\|y))) = (x\|y)$ | [12 → 6,4] |
| 160 | $((x\|(y\|y))\|y) = (y\|y)$ | [90 → 90] |
| 162 | $((x\|y)\|(y,1)) = y$ | [31 → 90] |
| 177,176 | $((x\|y)\|(x,1)) = x$ | [31 → 99] |
| 178 | $(x\|((((x\|y)\|z)\|y)\|(((x\|y)\|z)\|y))) = (x\|y)$ | [2 → 99] |
| 192 | $(x\|((y\|x),1)) = (y\|x)$ | [31 → 16] |
| 207,206 | $((x\|y)\|(1\|y)) = y$ | [92 → 90] |
| 208 | $((x\|y)\|(1\|x)) = x$ | [92 → 99] |
| 218 | $((1\|x)\|(y\|((x\|z)\|(x\|z)))) = x$ | [92 → 12] |
| 226 | $((x\|x)\|(y\|(1\|(x\|z)))) = x$ | [92 → 12] |
| 232,231 | $(1\|(1\|x)) = x$ | [92 → 93] |
| 234,233 | $(1\|(x,1)) = x$ | [31 → 93] |
| 291 | $((x\|(y\|z))\|(x\|(y\|y))) = ((x\|(y\|z))\|(x\|(y\|z)))$ | [18 → 35] |
| 296 | $((x\|(y\|z))\|(x\|(y\|z))) = ((x\|(y\|z))\|(x\|(y\|y)))$ | [291] |
| 300,299 | $((x\|y)\|(x\|y)) = (1\|(y\|x))$ | [6 → 102,94,94,94] |
| 302 | $((x\|(y\|z))\|(x\|(y\|y))) = (1\|((y\|z)\|x))$ | [296:300] |
| 328 | $((1\|x)\|(y\|(1\|(z\|x)))) = x$ | [218:300] |
| 335 | $(x\|(1\|(y\|((x\|y)\|z)))) = (x\|y)$ | [178:300] |
| 362 | $((x,1)\|(y\|(1\|(z\|x)))) = x$ | [122:300] |
| 369,368 | $((1\|(x\|y))\|z) = (y\|(1\|(x\|z)))$ | [103:300,300] |
| 382 | $(x\|(1\|(y\|(x\|y)))) = 1$ | [22:300] |
| 474 | $(x\|((y\|z)\|(x\|z))) = (x\|z)$ | [88 → 146] |
| 478 | $(x\|((y\|z)\|(y\|x))) = (x\|y)$ | [88 → 146] |
| 488 | $(x\|((x\|y)\|(y\|z))) = (x\|y)$ | [88 → 146] |
| 501 | $(x\|(x\|(x\|y))) = (x\|y)$ | [24 → 37,4] |
| 504,503 | $((x\|y)\|((x\|y)\|y)) = y$ | [162 → 37,91] |
| 505 | $((x\|y)\|((x\|y)\|x)) = x$ | [95 → 37,300,232,300,232,300,232,300,232,100] |
| 509 | $(x\|(x\|(y\|x))) = (x\|y)$ | [88 → 501] |
| 525 | $((x\|y)\|((y\|x)\|y)) = y$ | [88 → 503] |
| 531 | $(((x\|y)\|y)\|(x\|y)) = y$ | [88 → 503] |
| 552 | $(x\|((y\|y)\|(x\|z))) = (x\|((z\|z)\|(x\|y)))$ | [99 → 41] |
| 558 | $((x\|y)\|((z\|z)\|x)) = ((x\|y)\|(x\|((x\|y)\|z)))$ | [99 → 41] |
| 570 | $((x\|y)\|(x\|((x\|y)\|z))) = ((x\|y)\|((z\|z)\|x))$ | [558] |
| 591,590 | $(x\|((y\|x)\|x)) = (x\|y)$ | [88 → 509] |
| 637,636 | $((x\|y)\|(y\|x)) = ((x\|y),1)$ | [160 → 58,4,4,4,591,4] |
| 682 | $((x\|y)\|(y\|(y\|x))) = y$ | [88 → 525] |
| 709,708 | $(((x\|y)\|y)\|y) = (x\|y)$ | [531 → 505] |
| 939,938 | $(((x\|y)\|z)\|(1\|(y\|x))) = (x\|y)$ | [73 → 208,234,94,234] |
| 1014 | $((x\|y)\|((z\|((x\|y)\|y))\|y)) = y$ | [503 → 474,504] |
| 1022 | $(x\|((y\|z)\|(z\|x))) = (x\|z)$ | [88 → 474] |
| 1109 | $(x\|(y\|((z\|y)\|x))) = (x\|(z\|y))$ | [682 → 478] |
| 1119 | $(((x\|y)\|(x\|z))\|((z\|u)\|(z\|x))) = (((x\|y)\|(x\|z))\|z)$ | [478 → 478] |
| 1137 | $(x\|((y\|x)\|(y\|z))) = (x\|y)$ | [88 → 478] |
| 1146,1145 | $(((x\|y)\|(x\|z))\|z) = (z\|x)$ | [88 → 478] |
| 1147 | $(((x\|y)\|(x\|z))\|((z\|u)\|(z\|x))) = (z\|x)$ | [1119:1146] |
| 1166 | $(x\|((y\|y)\|(x\|z))) = (x\|((z\|x)\|(x\|y)))$ | [478 → 41] |
| 1191 | $(((x\|y)\|y)\|((x\|y)\|(y\|z))) = (x\|y)$ | [708 → 488,709] |
| 1673 | $(x\|(((y\|z)\|x)\|z)) = (x\|(y\|z))$ | [682 → 1137] |
| 1728 | $(((C\|(A\|A))\|(((((A\|B)\|C)\|C)\|A)\|(A\|D)))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$ | [88 → 104] |

| | | |
|---|---|---|
| 2276 | $(((C\|(A\|A))\|(((((B\|A)\|C)\|C)\|A)\|(A\|D)))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$ | [88 → 1728] |
| 3039 | $((x\|(y\|y))\|y) = (y\|y)$ | [35 → 192,25,36] |
| 3306 | $(((C\|(A\|A))\|((((C\|(B\|A))\|C)\|A)\|(A\|D)))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$ | [88 → 2276] |
| 3457 | $((x\|(1\|(y\|z)))\|(y\|y)) = y$ | [88 → 226] |
| 4471,4470 | $(((x\|y)\|(y\|z))\|y) = (1\|((y\|z)\|(x\|y)))$ | [3039 → 302,4,4,4] |
| 5717,5716 | $((x\|((y\|x)\|z))\|(u\|(1\|(y\|x)))) = (1\|(x\|((y\|x)\|z)))$ | [335 → 328,232] |
| 6399 | $(((C\|(A\|A))\|(((C\|(C\|(B\|A)))\|A)\|(A\|D)))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$ | [88 → 3306] |
| 11420 | $(x\|((y\|(x\|y))\|(x\|z))) = (x\|z)$ | [382 → 552,25,300,234] |
| 12749 | $((x\|y)\|((z\|((x\|y)\|z))\|x)) = x$ | [382 → 570,177,300,234] |
| 13756 | $(((C\|(A\|A))\|((A\|(C\|(C\|(B\|A))))\|(A\|D)))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$ | [88 → 6399] |
| 19707 | $(((C\|(A\|A))\|((A\|(C\|(C\|(B\|A))))\|(D\|A)))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$ | [88 → 13756] |
| 21759,21758 | $((x\|y)\|((z\|(y\|(x\|y)))\|y)) = y$ | [88 → 1014] |
| 24702,24701 | $(((x\|y)\|(y\|z)), 1) = (y\|((x\|y)\|(y\|z)))$ | [1109 → 1147,4471,939,637] |
| 26279,26278 | $(1\|(x\|((y\|x)\|(x\|z)))) = ((y\|x)\|(x\|z))$ | [1191 → 362,24702,5717] |
| 26698 | $(((C\|(A\|A))\|((D\|A)\|(A\|(C\|(C\|(B\|A))))))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$ | [88 → 19707] |
| 31493 | $((((D\|A)\|(A\|(C\|(C\|(B\|A)))))\|(C\|(A\|A)))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$ | [88 → 26698] |
| 33323 | $((((D\|A)\|(A\|(C\|(C\|(B\|A)))))\|(C\|(A\|A)))\|(A\|B)) \neq A \mid \$Ans(MOL_Sh)$ | [88 → 31493] |
| 46866 | $(x\|((y\|(y\|x))\|(x\|z))) = (x\|z)$ | [88 → 11420] |
| 48054,48053 | $((x\|y)\|((z\|(z\|(x\|y)))\|x)) = x$ | [88 → 12749] |
| 51292,51291 | $((x\|y)\|((z\|(z\|(x\|y)))\|y)) = y$ | [21758 → 46866,21759] |
| 51357 | $((x\|y)\|(y\|(z\|(z\|(x\|y))))) = y$ | [1022 → 46866,51292] |
| 51361 | $((x\|y)\|(x\|(z\|(z\|(x\|y))))) = x$ | [478 → 46866,48054] |
| 65437,65436 | $((x\|(y\|(y\|(z\|x))))\|(x\|z)) = x$ | [51357 → 51361,591] |
| 97556 | $((x\|((y\|z)\|(z\|u)))\|(z\|z)) = z$ | [1166 → 3457,26279] |
| 97921,97920 | $(((x\|y)\|(y\|z))\|(u\|(y\|y))) = (1\|((y\|z)\|(x\|y)))$ | [97556 → 1673,4471] |
| 98134 | $A \neq A \mid \$Ans(MOL_Sh)$ | [33323:97921,369,65437,207] |
| 98135 | $\$Ans(MOL_Sh)$ | [98134,1] |

# 4 The Computer Programs

See the primary paper [8].

# 5 Conclusion

See the primary paper [8].

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